# A HEIGHT-DIAMETER MODEL FOR BRUTIAN PINE (*PINUS BRUTIA* TEN.) PLANTATIONS IN SOUTHWESTERN TURKEY

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**Abstract.** In this study, models for the tree total height have been developed for brutian pine (*Pinus brutia* Ten.) stands in southwestern Turkey. For this purpose, 52 sample plots were measured. A total of 36 models that estimate the relationship between height and diameter in terms of stand variables (i.e. basal area, quadratic mean diameter, maximum diameter, dominant diameter, dominant height, arithmetic mean height, age, number of trees per hectare and site index), were fitted to correspond to 766 trees for non-linear regression procedures. Comparison of the models was carried out by using mean absolute error (MAE), maximum absolute error (MaxAE), root mean square error (RMSE), correlation coefficients (R), mean error (Bias) and the Akaike's information criterion (AIC). The most successful model among the 36 height-diameter models used was the Cox $II_a$ model. This model was followed by Cox $II_b$ and Sharma & Parton, respectively. As a result, the suggested model improves the accuracy of height prediction, ensures compatibility among the various estimates in a growth and yield model, and maintains projections within reasonable biological limits. Examples of applications of the selected generalized diameter-height models to the forest management are presented, namely how to use it to complete missing information from forest inventory and also showing how such an equation can be incorporated in a stand-level decision support system that aims to optimize the forest management for the maximization of wood volume production in southwestern Turkey brutian pine stands.
**Keywords:** *generalized height-diameter models, stand age, stand density, site index*

## Introduction

All models are an abstraction of reality that attempt to conceptualize key relationships of a system. Models can be both quantitative and conceptual in nature, but all models are integrators of multiple fields of knowledge. Forest growth and yield models are no different (Weiskittel et al., 2011). Total height is less frequently used in the development of forest models than diameter, as it is difficult and costly to measure, and consequently inaccurate measurements are often made (Sharma and Parton, 2007). When actual height measurements are not available, height-diameter functions can also be used to indirectly predict height growth (Larsen and Hann, 1987).

The relationship between tree height and diameter is one of the most important elements of forest structure. Many growth and yield models require height and diameter as basic input variables, with all or part of the tree height predicted from measured diameters (Wykoff et al., 1982; Huang et al., 2000).

Height-diameter relationships are applied to even-aged stands and can be fitted to linear functions, such as second-order polynomial equations, or more usually, to non-linear models (Colbert et al., 2002; Soares and Tomé, 2002; Castedo Dorado et al., 2006; Lootens et al., 2007). Model selecting a functional form for the height–diameter relationship, the following mathematical properties should be considered: (i) monotonic ascent, (ii) inflection point and (iii) horizontal asymptote (Lei and Parresol,


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2001). The number of parameters and their biological interpretation (e.g., asymptote, maximum or minimum growth rate) and satisfactory predictions of the height-diameter relationships are also important features (Peng, 1999).

A generalized height-diameter function estimates the specific relationship between individual tree heights and diameters using stand variables such as basal area per hectare, quadratic mean diameter, stands age, number of trees. The reason for using them is to avoid having to establish individual height-diameter relationships for every stand (Curtis, 1967). A wide variety of both local and generalized height-diameter models are available in the forestry literature (Huang et al., 2000; Soares and Tomé, 2002; López Sánchez et al., 2003; Temesgen and Gadow, 2004). Because of different geographical conditions in Turkey, the variety of tree species, habitat and stand structures is very high. However, equations reveal height-diameter relations for a limited number of different tree species and natural stands have been developed in Turkey (Sönmez, 2008; Mısır, 2010; Çatal, 2012; Diamantopoulou and Özçelik, 2012; Özçelik and Çapar, 2014). Generalized diameter-height models should be created to deal with pure brutian pine *(Pinus brutia Ten.)* plantation which was established 50 years ago. However, height-diameter models for brutian pine plantations are not yet available in Turkey.

Ecologically and economically, it is one of the most important forest tree species in Turkey. Brutian pine accounts for 25.1% of Turkey's total forest area, where it covers 5.6 million hectares. The species is considered fast growing and drought-tolerant with desirable wood characteristics. It is also widely used in reforestation and afforestation in Turkey (Anonymous, 2015). The aim of this study is to find an equation from selected generalized height-diameter models that could be used to predict the diameter-height relationship in artificial brutian pine stands in southwestern Turkey, by considering a number of stand variables (e.g., dominant diameter, dominant height, age, density, site index, etc.). The models divided into three groups and compare the models in three groups. These groups were the following i) diameter measurements, knowledge of stand age and number of trees per hectare, ii) measurements of diameter and height of sample trees, and ii) addition of measurements of stand age to the second group.

## Materials and methods

### *Data used*

Brutian pine is a characteristic species of the eastern Mediterranean and commonly found in fire-related ecosystems of the eastern Mediterranean region. It usually grows in pure stands and is valuable for its timber products as well as for soil stabilization and wildlife habitats. In Turkey, brutian pine forms extensive forests, especially in regions where the Mediterranean climate prevails.

This research was carried out in the region of southwestern Turkey, located, 50 km to the east of Burdur (*Fig. 1*).

The 650 ha study area is situated at 37°38'06'' N lat., 30°32'37'' E long., average slope 15°, predominantly north-facing aspect, 1,150 m asl. The soil is generally shallow or medium-deep, and stony, with a predominantly clay texture. Brutian pine plantations were established in 1974 using a spacing of 3 × 2 m.

Tree heights and diameters were measured in 52 sample plots established in pure and, even-aged artificial brutian pine stands in southwestern Turkey region. The plots were square or rectangular with areas varying between 400 and 1600 m$^2$. The number of


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trees per plot ranges between 32 and 115 depending on stocking. The sample plots were installed in order to provide the greatest variety of combinations of stand age, stand density degree and site index. In each sample plot, diameters at breast height of all trees were crosswise measured, using Haglöf calipers, to the nearest millimetre. Heights were measured using a Silva hypsometer to the nearest 0.1 m. In each sample plot 10-20 sample trees with different diameters and heights were chosen. Sample trees should not have any crown or stem damage.

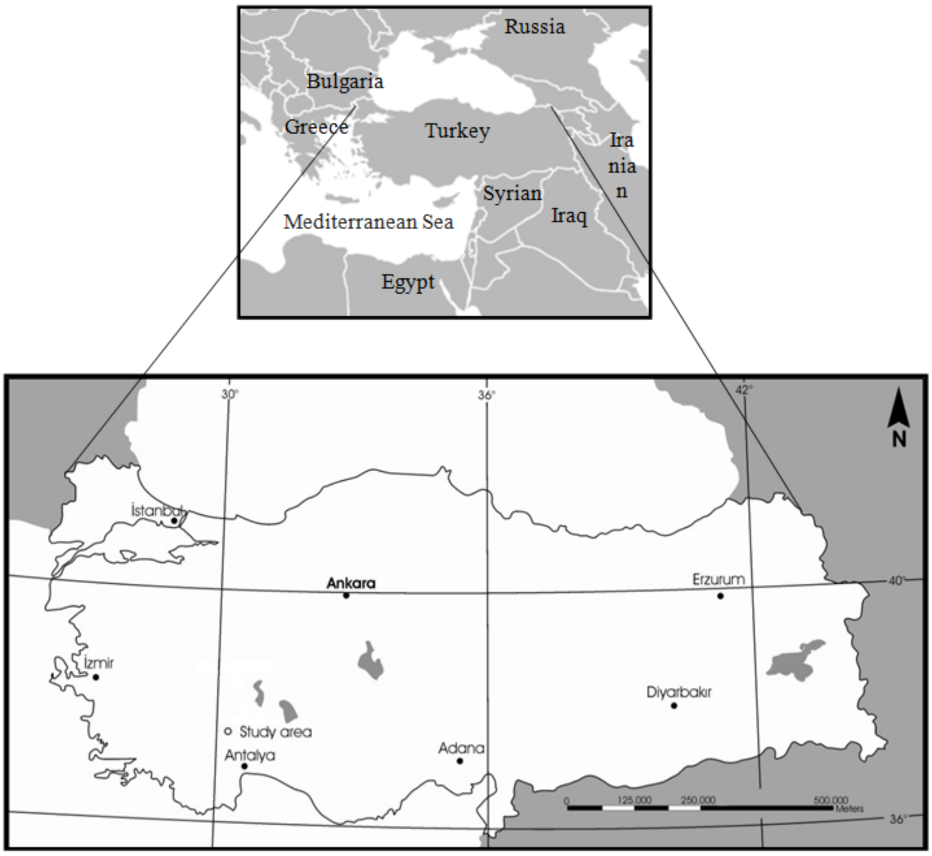


***Figure 1.*** *Location of sample plots in Turkey*

In addition the following stand variables were calculated based on the data collected in the plots: stand basal area, quadratic mean diameter, maximum diameter, dominant diameter, dominant height, stand mean height, stand age (it was calculated from the year of planting), stand density and site index, defined as stand dominant height at 30 years of age and determined from the site index curves available for this species in the region (Usta, 1991).

### *Models analysed*

A large number of generalized height-diameter models have been discussed in the forestry literature, many of which been developed for modelling the relationship between tree height and diameter at breast height by additional stand and site variables. In the present study, we have considered the most commonly used 36 generalized height-diameter equations (*Table 1*).


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***Table 1.*** *Generalized height-diameter models evaluated*

| The first group model (Group model 1) | Author (s) | Model |
|---|---|---|
| $h = 10^{(a_0 + a_1 \frac{1}{d} + a_2 \frac{1}{t} + a_3 \frac{1}{d_g t})}$ | Curtis (1967) | 1 |
| $h = e^{(a_0 + a_1 \ln d_g + a_2 \ln N + a_3 \sqrt{d})}$ | Cox I (1994) | 2 |
| $h = 1.30 + 10^{(a_0 + a_1 \frac{1}{d} + a_2 \frac{1}{\sqrt{t}} + a_3 \frac{1}{d\sqrt{t}} + a_4 \frac{\log N}{\sqrt{t}})}$ | Clutter and Allison (1974) | 3 |
| **The second group model (Group model 2)** | **Author (s)** | **Model** |
| $h = 1.30 + \left[ a_0 (\frac{1}{d} - \frac{1}{D_0}) + (\frac{1}{H_0 - 1.3})^{1/3} \right]^{-3}$ | Mǿnnes (1982) | 4 |
| $h = 1.30 + (H_0 - 1.3)(\frac{d}{D_0})^{a_0}$ | Canadas et al. I (1999) | 5 |
| $h = 1.30 + \dfrac{d}{\frac{D_0}{H_0 - 1.3} + a_0 (D_0 - d)}$ | Canadas et al. II (1999) | 6 |
| $h = 1.30 + (H_0 - 1.3) \dfrac{1 - e^{a_0 d}}{1 - e^{a_0 D_0}}$ | Canadas et al. III (1999) | 7 |
| $h = 1.30 + \left[ a_0 (\frac{1}{d} - \frac{1}{D_0}) + (\frac{1}{H_0 - 1.3})^{1/2} \right]^{-2}$ | Canadas et al. IV (1999) | 8 |
| $h = 1.30 + (H_0 - 1.3)\, e^{a_0 (1 - \frac{d_g}{d}) + a_1 (\frac{1}{d_g} - \frac{1}{d})}$ | Gaffrey (1988) | 9 |
| $h = 1.30 + (H_0 - 1.3)\,[1 + a_0 (H_0 - 1.3)(\frac{1}{d} - \frac{1}{D_0})]^{-1}$ | Prodan (1968) | 10 |
| $h = 1.30 + H_0 (1 + a_0 + a_1 H_0 + a_2 d_g)\, e^{a_3 H_0}\, (1 - e^{a_4 \frac{d}{H_0}})$ | Soares and Tome (2002) | 11 |
| $h = 1.30 + (H_m - 1.3)\, e^{a_0 (1 - \frac{d}{d_g})}\, e^{a_1 (\frac{d}{d_g} - \frac{1}{d})}$ | Sloboda et al. (1993) | 12 |
| $h = H_0 (1 + a_0\, e^{a_1 H_0})\,(1 - e^{\frac{-a_2 d}{H_0}})$ | Harrison et al. (1986) | 13 |
| $h = 1.30 + \dfrac{a_0 {H_0}^{a_1} (H_0 - 1.3)}{(\left\lvert \frac{D_0 - d}{d} \right\rvert)^{a_2}}$ | Castedo Dorado et al. (2001) | 14 |
| $h = a_0 H_0 (1 - e^{\frac{-a_1 d}{d_g}})^{a_2}$ | Pienaar et al.$_a$ (1990) | 15 |
| $h = a_0 H_0 (1 - e^{\frac{-a_1 d}{D_0}})^{a_2}$ | Pienaar et al.$_b$ (1990) | 16 |
| $h = 1.30 + (H_m - 1.3)\, e^{a_0 (1 - \frac{d}{D_0})}\, e^{a_1 (\frac{d}{D_0} - \frac{1}{d})}$ | Sloboda et al. (1993) | 17 |
| $h = 1.30 + a_0 {H_0}^{a_1} d^{{a_2 H_0}^{a_3}}$ | Hui and Gadow (1993) | 18 |


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| | | |
|---|---|---|
| $h=1.30+(a_0+a_1H_0-a_2d_g)\,e^{(\frac{a_3}{d})}$ | Mirkovich (1958) | 19 |
| $h=1.30+(a_0+a_1G+a_2H_0)\,e^{(a_3\frac{1}{d})}$ | Ademe et al. (2008) | 20 |
| $h=1.30+(a_0+a_1H_0-a_2d_g)\,e^{(\frac{a_3}{\sqrt{d}})}$ | Schröder and Álvarez González I (2001) | 21 |
| $h=H_m\left[a_0+a_1H_m+a_2\frac{H_m}{d_g}+a_3d+a_4\frac{\frac{N}{d_g(H_md_g)}}{d}\right]$ | Cox III$_a$ (1994) | 22 |
| $h=1.30+(a_0+a_1H_0-a_2d_g+a_3G)\,e^{(\frac{-a_4}{\sqrt{d}})}$ | Schröder and Álvarez González II (2001) | 23 |
| $h=a_0+a_1H_m+a_2d_g^{0.95}+a_3e^{-0.08d}+a_4H_m^3e^{-0.08d}+a_5d_g^3e^{-0.08d}$ | Cox II$_a$ (1994) | 24 |
| $h=a_0+a_1H_m+a_2d_g+a_3e^{a_4d}+a_5H_m{}^{a_6}e^{a_4d}+a_7d_g{}^{a_8}e^{a_4d}$ | Cox II$_b$ (1994) | 25 |
| $h=1.30+a_0G^{a_1}(1-e^{(-a_2N^{a_3}d)})$ | Sharma and Zhang (2004) | 26 |
| $h=1.30+a_0H_0{}^{a_1}(1-e^{(-a_2(\frac{N}{G})^{-a_3}\,d)})^{a_4}$ | Sharma and Parton (2007) | 27 |
| $h=1.30+a_0\;H_0{}^{a_1}(1-e^{(-a_2D_0{}^{a_3}d)})^{a_4}$ | Richards (1959) | 28 |
| $h=1.30+a_0(H_0-1.30)^{a_1}\;e^{(-a_2d^{-a_3}+a_4G)}$ | Budhathoki et al.(2008) | 29 |
| $h=\left[1.30^{a_0}+\left(H_0^{a_0}-1.30^{a_0}\right)*\frac{1\text{-}e^{-a_1d}}{1\text{-}e^{-a_1D_0}}\right]^{\frac{1}{a_0}}$ | Castedo Dorado et al. (2001) | 30 |
| **The third group model (Group model 3)** | **Author (s)** | **Model** |
| $h=H_0e^{(a_0+a_1H_0+a_2\frac{N}{1000}+a_3t)(\frac{1}{d}-\frac{1}{D_0})}$ | Tome (1989) | 31 |
| $h=e^{(a_0+a_1SI+a_2\frac{N}{1000}+a_3\frac{1}{t}+a_4\frac{1}{d})}$ | Bennet and Clutter (1968) | 32 |
| $h=1.30+\frac{H_0}{e^{a_0+(\frac{1}{d}-\frac{1}{D_{\max}})\,(a_1+a_2\ln N+a_3\frac{1}{t}+a_4\ln H_0)}}$ | Lenhart (1968) | 33 |
| $h=a_0H_0{}^{a_1}10^{(\frac{a_2}{t}+(\frac{1}{d}-\frac{1}{D_{\max}})\,(a_3+a_4\frac{\log N}{t})}$ | Amateis et al. (1995) | 34 |
| $h=e^{(a_0+a_1\ln H_0+a_2\frac{1}{t}+a_3\frac{\ln N}{d}+a_4\frac{1}{dt}+a_5\frac{1}{d})}$ | Burkhart and Strub (1974) | 35 |
| $h=a_0H_0{}^{a_1}G^{a_2}N^{a_3}e^{(\frac{a_4}{t}+\frac{a_5}{d})}$ | Pascoa (1987) | 36 |

The terminology used in the up models is as follows: d = diameter at breast height over bark (cm), t = age of stand, $d_g$ = quadratic mean diameter of stand (cm), G = basal area of stand ($m^2$/ha), $D_{max}$ = maximum diameter of stand (cm), $D_o$ = dominant diameter of stand (cm), $H_m$ = mean height of stand (m), $H_o$ = dominant height of stand (m), N = number of trees in stand (stems/ha), SI = site index (m), log = common logarithm (base 10), ln = natural logarithm (base e = 2,718), $a_0$, $a_1$… = regression coefficients


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The models were classified in three groups according to the sampling effort (Sanchez et al., 2003). These groups; i) *low sampling effort models;* including measurements of diameter and knowledge of stand age, ii) *medium sampling effort models*, including measurements of diameter and heights of sample tree, iii) *high sampling effort models*, including knowledge or measurements of stand age as well.

### *Statistical analysis*

In this study the models described above are non-linear; therefore model fitting was carried out with non-linear regression (NLIN) procedure of SPSS statistical analysis software package. The initial values of parameters were obtained by starting the iterative procedure also used by other authors in similar studies (Castedo Dorado et al., 2006; Özçelik and Çapar, 2014; Ahmadi et al., 2016).

Comparison of estimation of models was based on graphical and numerical analysis of residuals and six goodness of fit statistics: mean absolute error (MAE), which expresses the average of absolute errors between forecast and actual value; maximum absolute error (MaxAE), which maximum absolute value for prediction values; root mean square error (RMSE), which analyses the precision of estimations; correlation coefficients (R), which reflect the total variability that is explained by the model considering the total number of parameters to be estimated; mean error (Bias), which average error for estimated values, and the Akaike's information criterion (AIC), which is an index that is used to select the best model. These evaluation statistics are defined as (*Eqs. 1-6*):
Mean absolute error:

$$MAE = \frac{\sum_{i=1}^{n} \left| h_i - \hat{h}_i \right|}{n} \quad \text{(Eq.1)}$$

Maximum absolute error:

$$MaxAE = Max(\left| h_i - \hat{h}_i \right|) \quad \text{(Eq.2)}$$

Root mean square error:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (h_i - \hat{h}_i)^2}{n-k}} \quad \text{(Eq.3)}$$

Correlation coefficients:

$$R = \frac{\sum_{i=1}^{n} (h_i - \bar{h}_i) * (\hat{h}_i - \bar{h}_i)}{\sqrt{\sum_{i=1}^{n} (h_i - \bar{h}_i)^2} * \sqrt{\sum_{i=1}^{n} (\hat{h}_i - \bar{h}_i)^2}} \quad \text{(Eq.4)}$$


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Mean error:

$$Bias = \frac{\sum_{i=1}^{n}(h_i - \hat{h}_i)}{n} \qquad \text{(Eq.5)}$$

Akaike's information criterion:

$$AIC = n * ln\left(RMSE\right) + 2 * p \qquad \text{(Eq.6)}$$

where $h_i$ = observed height, $\hat{h}_i$ redicted height, $\bar{h}_i$ ean of observed heights, $n$ = number of observations in dataset and $k$ = number of estimated parameters.

## Results and discussion

### *Data summary*

Approximately 80% (42 sample plots) of sample plots data were used to develop model and remaining 20% (10 sample plots) were used to test developed models. The dataset for test of developed models was intended to obtain a measure of the adequacy of the calibration from different sampling stands. Since the data set is large enough, this proportions used is unlikely to reduce the precision of the parameter estimates compared with those obtained with the model built from the entire dataset in forestry research or data mining (Soares and Tomé, 2002; Castedo Dorado et al., 2006). The mean, minimum and maximum values and standard deviations of stand variables are shown in *Table 2*.

***Table 2.*** *Characteristics of the fitting and evaluation data set*

| Variables | Fitting data set (n = 794) | | | | Evaluation data set (n = 241) | | | |
|---|---|---|---|---|---|---|---|---|
| | **Mean** | **Minimum** | **Maximum** | **Standard deviation** | **Mean** | **Minimum** | **Maximum** | **Standard deviation** |
| d (cm) | 21.2 | 12.1 | 34.7 | 3.2 | 21.5 | 11.0 | 33.5 | 3.6 |
| h (m) | 9.9 | 5.9 | 19.0 | 1.8 | 10.5 | 5.5 | 16.0 | 2.0 |
| A (yr) | 36 | 32 | 41 | 2.5 | 37 | 32 | 41 | 2.8 |
| dg (cm) | 20.9 | 18.1 | 25.3 | 1.6 | 21.3 | 18.0 | 26.3 | 2.0 |
| G ($m^2 ha^{-1}$) | 27.6 | 17.1 | 42.5 | 4.6 | 28.7 | 20.6 | 46.4 | 6.0 |
| N (trees $ha^{-1}$) | 814 | 448 | 1136 | 136 | 805 | 624 | 1104 | 140.8 |
| $H_o$ (m) | 10.6 | 8.0 | 17.2 | 1.6 | 11.6 | 9.4 | 15.3 | 2.0 |
| $D_o$ (cm) | 25.1 | 21.9 | 30.9 | 1.9 | 25.6 | 21.0 | 31.2 | 2.4 |
| $D_{max}$ (cm) | 27.3 | 23.7 | 34.7 | 2.3 | 27.3 | 22.3 | 34.6 | 2.9 |
| $H_m$ (m) | 9.8 | 7.5 | 15.3 | 1.4 | 10.5 | 8.4 | 13.5 | 1.5 |
| SI (m) | 9.5 | 7.5 | 15.7 | 1.4 | 10.3 | 8.9 | 13.6 | 1.7 |

The terminology used in the table is as follows: d = diameter at breast height over bark (cm), A = age of stand (yr), $d_g$ = quadratic mean diameter of stand (cm), G = basal area of stand ($m^2$/ha), $D_{max}$ = maximum diameter of stand (cm), $D_o$ = dominant diameter of stand (cm), h = height of trees (m), $H_m$ = mean height of stand (m), $H_o$ = dominant height of stand (m), N = number of trees in stand (trees/ha), SI = site index (m), n = number of sampling trees


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Relationship between height and diameter for data fitting model, validation model and all data are shown in *Figure 2*.

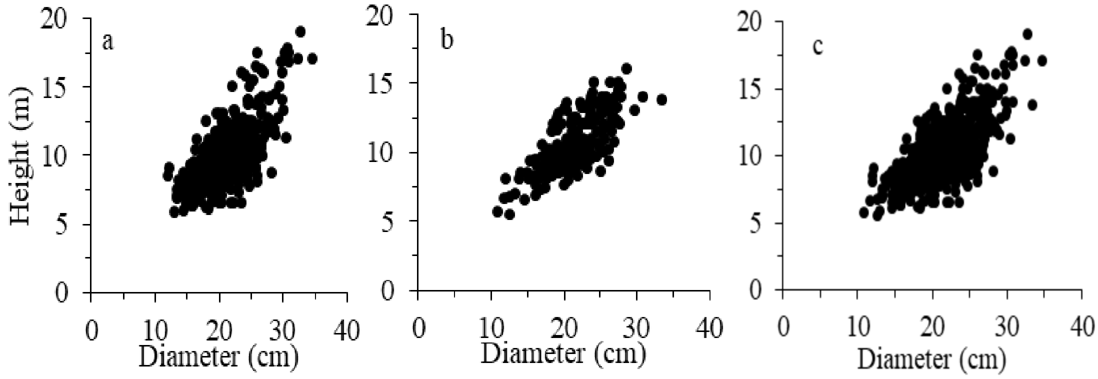


***Figure 2.*** *Relationship between diameter and height for data fitting model (a), data validation model (b) and all data (c)*

## Model fitting

Modelling for biological systems sense is an important tool. Modelling is the process of defining a system's change with equations (Weiskittel et al., 2011). It is therefore important to accurately determine the components of system during modelling and to select the correct equation to describe this system. In our study, it was tried to explain the change of tree height in relation with diameter at breast height according to the regression models in southwestern Turkey. The parameter values for all equations are included in *Table 3*.

The model parameters for all the tested models were found to be significant at the significance level of 0.001. In order to find out which model was more successful in explaining height-diameter relation, a ranking was made for all models according to the specified criteria and the results were shown below. In this ranking method, numerical values were given starting from the smallest MAE, MaxAE, RMSE, ME, AIC ones and for the R value, starting with the highest one. When the ranking values obtained for each model were collected, the model with the smallest value was considered as the best one (*Table 4*).

In this study were found to be similar with the model results of the previous studies (Sanchez et al., 2003; Castedo Dorado et al., 2006). In terms of group averages, the third group of equations was found to be more successful. But, the most successful model among the 36 height-diameter models used was the Cox $II_a$ model, followed by Cox $II_b$ and Sharma and Parton, respectively. Curtis, Gaffrey & Sharma and Zhang models have been not suitable for this region.

The results of fitting and cross-validation for the models of group 1 were the poorest. In this respect, a number of studies showed that adding stand variables to the height-diameter equation and using the generalized height-diameter models increased the precision (Sharma and Parton, 2007; Krisnawati et al., 2010; Temesgen et al., 2014). These stand variables mentioned in the literature are dominant height, stand basal area, maximum diameter, stand age, number of trees per hectare, stand density. The statistics and coefficients according to the studied model were found to be similar to the results of the previous model studies (Larsen and Hann, 1987; Colbert et al., 2002). The inclusion


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of basal area and $d_g$ into the base height-diameter function increased the accuracy of prediction (Temesgen and Gadow, 2004).

***Table 3.*** *Parameters of non-linear regression models*

| Model no | Parameters | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **$a_0$** | **$a_1$** | **$a_2$** | **$a_3$** | **$a_4$** | **$a_5$** | **$a_6$** | **$a_7$** | **$a_8$** |
| 1 | 1.716443 | -5.306083 | -15.181403 | -0.022643 | - | - | - | - | - |
| 2 | -4.159129 | 0.960891 | 0.338363 | 0.275196 | - | - | - | - | - |
| 3 | 3.834179 | -41.321130 | -19.480656 | 207.082437 | 1.393833 | - | - | - | - |
| 4 | 1.846925 | - | - | - | - | - | - | - | - |
| 5 | 0.526950 | - | - | - | - | - | - | - | - |
| 6 | -0.048474 | - | - | - | - | - | - | - | - |
| 7 | -0.053122 | - | - | - | - | - | - | - | - |
| 8 | 1.924790 | - | - | - | - | - | - | - | - |
| 9 | -0.005893 | 10.695102 | - | - | - | - | - | - | - |
| 10 | 0.283957 | - | - | - | - | - | - | - | - |
| 11 | -353.995785 | 7.157280 | 5.451819 | 0.129807 | 0.000621 | - | - | - | - |
| 12 | -1.851475 | -0.989978 | - | - | - | - | - | - | - |
| 13 | 0.106958 | 1.500000 | 0.468510 | - | - | - | - | - | - |
| 14 | 0.966326 | -0.051939 | 0.037236 | - | - | - | - | - | - |
| 15 | 1.165160 | 2.226022 | 1.982192 | - | - | - | - | - | - |
| 16 | 1.298927 | 1.668149 | 1.167713 | - | - | - | - | - | - |
| 17 | -22.383484 | -6.374520 | - | - | - | - | - | - | - |
| 18 | 6.314068 | -0.311706 | 0.424369 | 0.074948 | - | - | - | - | - |
| 19 | 7.913832 | 1.473407 | 0.358958 | -13.137764 | - | - | - | - | - |
| 20 | 2.261735 | -0.016805 | 1.239617 | -11.641012 | - | - | - | - | - |
| 21 | 15.323304 | 2.724760 | 0.677738 | -5.761236 | - | - | - | - | - |
| 22 | 0.468122 | -0.045892 | 0.929943 | 0.025909 | 0.471646 | - | - | - | - |
| 23 | 15.386189 | 2.750300 | 0.674077 | -0.015238 | 5.759656 | - | - | - | - |
| 24 | 1.651100 | 1.043897 | 0.073152 | -5.834839 | -0.000625 | -0.001224 | - | - | - |
| 25 | -122.918442 | 0.975019 | -0.790733 | 121.929973 | 0.001889 | $5x10^{-9}$ | 6.477289 | 0.813999 | 0.894380 |
| 26 | 3.178352 | 0.562636 | 0.115088 | -0.223862 | | - | - | - | - |
| 27 | 1.122581 | 0.988632 | 0.026346 | -0.374752 | 1.972110 | - | - | - | - |
| 28 | 1.072885 | 1.031935 | 1.141993 | -0.850653 | 1.487636 | - | - | - | - |
| 29 | 0.016350 | 0.770381 | -3.156851 | -0.121544 | -0.001077 | - | - | - | - |
| 30 | -0.000691 | 0.114182 | - | - | - | - | - | - | - |
| 31 | -2.608873 | -0.804187 | 0.196157 | 0.037243 | | - | - | - | - |
| 32 | 2.494066 | 0.060584 | 0.101072 | -13.008971 | -10.368192 | - | - | - | - |
| 33 | 0.073700 | 33.485427 | -3.034925 | -11.024664 | -0.154562 | - | - | - | - |
| 34 | 1.224536 | 0.926236 | 0.319181 | -7.179243 | 29.976501 | - | - | - | - |
| 35 | 0.542895 | 0.804716 | 13.096415 | 1.259923 | -162.752624 | -14.715665 | - | - | - |
| 36 | 1.150484 | 0.893337 | -0.206565 | 0.188713 | 0.415724 | -11.460517 | - | - | - |

$a_0$, $a_1$… = regression coefficients

The values of statistics of the models included in group model 2 show that the second modification of Cox $II_a$ is the equation that most accurately estimates height. The best equation was found in the second group because of a low variation in stand age. When the stand age variation was high, the group model 2 was more successful than group model 3.


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The models of Cox $II_b$ and Sharma & Parton also fit well to the data in *Table 4*. The advantage of these models was that they were functions of simple equation, although the bias and MSE were slightly higher than those of the modified versions of the Cox $II_a$ model.

***Table 4.*** *Performance criteria for generalized height-diameter models for the fitting data*

| Model no | Performance criteria | | | | | | |
|---|---|---|---|---|---|---|---|
| | **MAE** | **MaxAE** | **RMSE** | **R** | **ME** | **AIC** | **Rank** |
| 1 | 1.23352 (36) | 4.70653 (35) | 1.54613 (36) | 0.51256 (36) | -0.91648 (36) | 353.990 (36) | 36 |
| 2 | 0.90645 (34) | 3.97724 (33) | 1.15783 (33) | 0.76586 (33) | 0.00338 (16) | 124.359 (33) | 30 |
| 3 | 0.86868 (32) | 5.37752 (36) | 1.16102 (34) | 0.76472 (34) | 0.00182 (14) | 128.543 (35) | 32 |
| 4 | 0.61337 (17) | 2.90029 (19) | 0.78395 (15) | 0.89985 (15) | 0.08043 (29) | -191.270 (13) | 20 |
| 5 | 0.61940 (21) | 2.94987 (24) | 0.79151 (19) | 0.89780 (20) | 0.09082 (35) | -183.647 (17) | 24 |
| 6 | 0.63727 (26) | 2.98865 (26) | 0.81630 (25) | 0.89090 (25) | 0.15062 (31) | -159.161 (24) | 27 |
| 7 | 0.61708 (19) | 2.90005 (18) | 0.78865 (17) | 0.89858 (17) | 0.08608 (30) | -186.522 (15) | 21 |
| 8 | 0.61260 (15) | 2.91290 (22) | 0.78312 (14) | 0.90007 (14) | 0.07878 (28) | -192.109 (12) | 17 |
| 9 | 0.89279 (33) | 3.73911 (30) | 1.11244 (32) | 0.78572 (32) | -0.07513 (34) | 88.605 (32) | 35 |
| 10 | 0.85097 (31) | 3.96399 (32) | 1.09746 (31) | 0.79191 (31) | -0.53040 (33) | 75.841 (31) | 33 |
| 11 | 0.68984 (28) | 3.03966 (28) | 0.88701 (28) | 0.87043 (27) | 0.05039 (25) | -85.200 (28) | 28 |
| 12 | 0.56765 (3) | 2.65307 (9) | 0.71431 (2) | 0.91773 (3) | 0.05888 (26) | -263.132 (2) | 6 |
| 13 | 0.63052 (23) | 2.79037 (16) | 0.80420 (22) | 0.89458 (22) | -0.00365 (17) | -167.018 (22) | 23 |
| 14 | 0.73644 (30) | 3.66945 (29) | 0.94454 (29) | 0.85117 (29) | 0.00550 (19) | -39.304 (30) | 29 |
| 15 | 0.61041 (13) | 2.75054 (12) | 0.78863 (16) | 0.89886 (16) | 0.02120 (24) | -182.542 (18) | 14 |
| 16 | 0.60687 (11) | 2.93672 (23) | 0.77907 (11) | 0.90143 (11) | 0.01516 (23) | -192.226 (10) | 12 |
| 17 | 0.73333 (29) | 4.35856 (34) | 0.94479 (30) | 0.85088 (30) | 0.44719 (32) | -41.093 (29) | 31 |
| 18 | 0.68532 (27) | 2.88025 (17) | 0.88680 (27) | 0.87032 (28) | -0.00723 (20) | -87.388 (27) | 26 |
| 19 | 0.59900 (6) | 2.58431 (5) | 0.77400 (7) | 0.90290 (7) | 0.00023 (3) | -195.410 (5) | 4 |
| 20 | 0.62974 (22) | 2.69422 (10) | 0.80497 (23) | 0.89451 (23) | 0.00021 (1) | -164.259 (23) | 15 |
| 21 | 0.59904 (7) | 2.62233 (8) | 0.77492 (8) | 0.90266 (9) | -0.00045 (6) | -194.466 (8) | 7 |
| 22 | 0.57292 (4) | 2.60799 (7) | 0.72224 (4) | 0.91615 (4) | 0.00141 (13) | -248.366 (3) | 5 |
| 23 | 0.59916 (8) | 2.60273 (6) | 0.77526 (9) | 0.90270 (8) | -0.00046 (7) | -192.118 (11) | 8 |
| 24 | 0.56047 (1) | 2.36446 (1) | 0.70628 (1) | 0.92008 (1) | -0.00044 (5) | -264.108 (1) | 1 |
| 25 | 0.56484 (2) | 2.49484 (2) | 0.71598 (3) | 0.91810 (2) | -0.00281 (15) | -247.278 (4) | 2 |
| 26 | 0.91127 (35) | 3.89076 (31) | 1.16327 (35) | 0.76332 (35) | 0.00788 (21) | 128.081 (34) | 34 |
| 27 | 0.59885 (5) | 2.55334 (4) | 0.77289 (5) | 0.70332 (5) | 0.00037 (4) | -194.549 (6) | 3 |
| 28 | 0.60058 (9) | 2.95984 (25) | 0.77290 (6) | 0.90332 (6) | -0.00022 (2) | -194.539 (7) | 9 |
| 29 | 0.63466 (24) | 2.73507 (11) | 0.80966 (24) | 0.89335 (24) | 0.00127 (11) | -157.646 (25) | 22 |
| 30 | 0.60831 (12) | 2.90991 (21) | 0.78097 (12) | 0.90079 (12) | 0.01201 (22) | -192.292 (9) | 11 |
| 31 | 0.61129 (14) | 2.90111 (20) | 0.78283 (13) | 0.90055 (13) | 0.06951 (27) | -186.403 (16) | 16 |
| 32 | 0.63609 (25) | 3.01366 (27) | 0.81981 (26) | 0.89049 (26) | -0.00138 (12) | -147.754 (26) | 25 |
| 33 | 0.61363 (18) | 2.78609 (15) | 0.79100 (18) | 0.89848 (18) | -0.00409 (18) | -176.159 (19) | 19 |
| 34 | 0.61299 (16) | 2.75859 (13) | 0.79161 (20) | 0.89832 (19) | 0.00094 (9) | -175.547 (20) | 13 |
| 35 | 0.61730 (20) | 2.78036 (14) | 0.79496 (21) | 0.89754 (21) | 0.00069 (8) | -170.194 (21) | 18 |
| 36 | 0.60169 (10) | 2.50238 (3) | 0.77717 (10) | 0.90232 (10) | 0.00118 (10) | -188.160 (14) | 10 |

MAE = mean absolute error, MaxAE = maximum absolute error, RMSE = root mean square error, R = correlation coefficients, Bias = mean error, AIC = Akaike's information criterion and rank = numerical values in ranking method


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Errors of actual heights versus heights predicted in the fitting phase of the Cox $II_a$, Cox $II_b$ and Sharma & Parton models are shown in *Figure 3*. There was no reason to reject the hypotheses of normality, homogeneity of variance and independence of residuals.

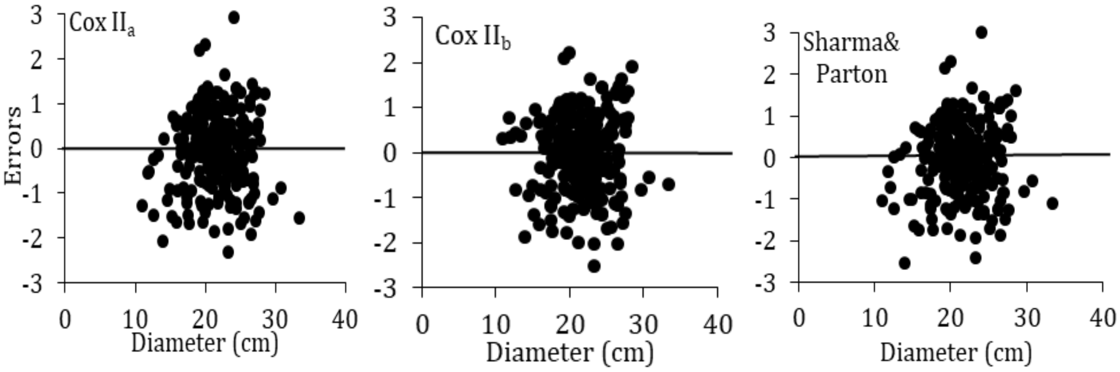


***Figure 3.*** *Errors actual heights versus predicted values in the fitting phase for the of Cox $II_a$, Cox $II_b$, Sharma & Parton models*

In general, it was found that the error amounts show an increase in successful models due to the increase in height values (Ahmadi et al., 2013; Özçelik and Çapar, 2014). The amount of error in our work did not increase, but decreased because this forest is plantation and trees have got similar height. It can be said that the variation with respect to the error distributions obtained with the generalized height-diameter models is relatively constant (from -2 m to +2 m).

In this study, when it was decided whether a model is successful, it is required that the amount of error was small, and that it has a certain and constant variance in the errors were obtained. Tree height changed from 5.9 to 19.0 m in this study (in *Table 2*). In predicted of a tree height error about 2 meter was small. Cox IIa, Cox IIb, and Sharma & Parton models were found successful in this respect.

In describing the diameter-height relationship, group 3 models including the stand age was more successful (Sánchez et al., 2003). However, results of group 2 models were found as the best models because of data were taken from artificial stands in this study.

Cox $II_a$, Cox $II_b$, and Sharma & Parton models could offer a balance between the accuracy of model and sampling effort, because the value of stand age was not including for plantation in Group 2 models.

Observed heights versus the predicted heights in the cross-validation of this model are shown in *Figure 4*. The performance criterion to evaluate the behaviour of model was the determination coefficient of the straight line fitted between the observed and predicted heights. *Figure 4* shows no tendency toward the overestimation or underestimation of height values.

For the tested models, the results obtained using the independent data set is given in *Figure 4*. The most similar results of Cox $II_a$, Cox $II_b$ and Sharma & Parton models are shown in *Figure 4*. Relatively similar results were obtained for same models (Sánchez et al., 2003). The overlap ratio of the predicted height values with the measured height values does not increase as the height value increases.


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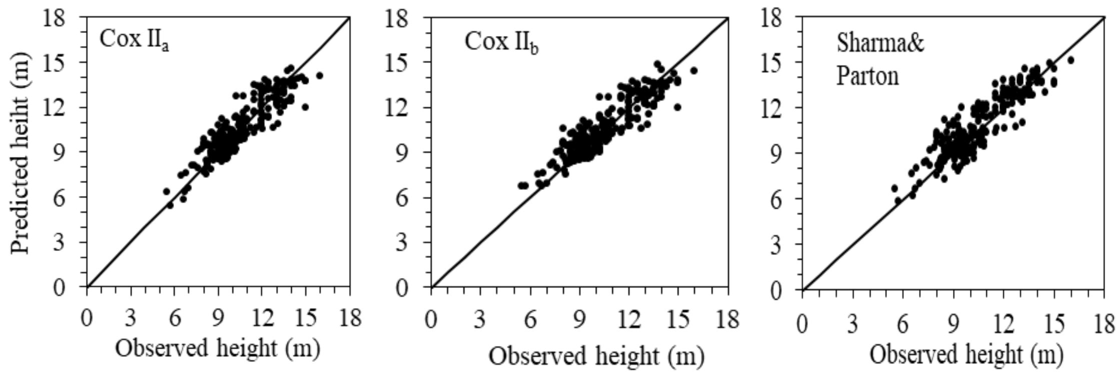


***Figure 4.*** *Observed heights versus predicted heights in the cross-validation for the Cox II$_a$, Cox II$_b$ and Sharma & Parton models*

As can be seen from *Table 5*, the tested height-diameter models were not very different from the model development data.

***Table 5.*** *Criterion values for successful models with independent data set*

| Model no | Performance criteria | | | | | | |
|---|---|---|---|---|---|---|---|
| | **MAE** | **MaxAE** | **RMSE** | **R** | **ME** | **AIC** | **Rank** |
| 24 | 0.68882 (3) | 3.00290 (5) | 0.87938 (2) | 0.89799 (3) | -0.04821 (1) | -18.9777 (2) | 2 |
| 25 | 0.67832 (1) | 2.99999 (4) | 0.00113 (3) | 0.89895 (2) | -0.05729 (3) | -12.4986 (4) | 3 |
| 27 | 0.71003 (5) | 2.73425 (2) | 0.91893 (5) | 0.88752 (5) | -0.19489 (5) | -10.3754 (5) | 5 |
| 19 | 0.70434 (4) | 2.66425 (1) | 0.90841 (4) | 0.88974 (4) | -0.19010 (4) | -15.1503 (3) | 4 |
| 22 | 0.68160 (2) | 2.93107 (3) | 0.87081 (1) | 0.89963 (1) | -0.05696 (2) | -23.3373 (1) | 1 |

MAE = mean absolute error, MaxAE = maximum absolute error, RMSE = root mean square error, R = correlation coefficients, Bias = mean error, AIC = Akaike's information criterion and rank = numerical values in ranking method

Finally, the most successful models were used for all sample plot data. Regression coefficients and statistics of these models are shown in *Table 6*. These parameters can use to estimate of diameter-height relationship for artificial brutian pine in southwestern Turkey.

***Table 6.*** *Regression coefficients and statistics obtained for the d-h models using the entire data set*

| Model no | Parameters | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **$a_0$** | **$a_1$** | **$a_2$** | **$a_3$** | **$a_4$** | **$a_5$** | **$a_6$** | **$a_7$** | **$a_8$** |
| 24 | 4.527680 | 1.217846 | -0.178316 | -9.800583 | -0.003507 | -0.000524 | - | - | - |
| 25 | -122.56039 | 0.983721 | -0.711003 | 121.203832 | 0.002021 | $4.8x10^{-51}$ | 42.055083 | 0.903834 | 0.812250 |
| 27 | 1.426453 | 0.909989 | 0.028413 | -0.304526 | 1.710843 | - | - | - | - |
| 19 | 8.405537 | 1.393674 | 0.322209 | -13.749983 | 1.970000 | - | - | - | - |
| 22 | 0.431054 | -0.045442 | 0.957580 | 0.025728 | 0.006561 | - | - | - | - |

$a_0$, $a_1$… = regression coefficients


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## Conclusions

In this study, 36 height-diameter models were calibrated and tested for brutian pine plantations in southwestern Turkey. The best predictions of height were obtained by the Cox $II_a$ model, which used diameter (d), quadratic mean diameter ($d_g$), and stand mean height ($H_m$) as independent variables. In this model, provides little effort has been made to model the height-diameter relationship in uneven-aged stands with generalized height-diameter functions. In addition, group 2 models should be used instead of group 3 models in the artificial stands.

The inclusion of stand mean height or of stand dominant height as an independent variable in the height-diameter equations seems to be necessary in order to achieve acceptable predictions. This requires the measurement of at least one sample of heights for the practical application of the equation. The inclusion of $d_g$ into the base height-diameter model increased the accuracy of prediction.

As a result, examples of applications of the selected generalized height-diameter models to the forest management are presented, namely how to use it to complete missing information from forest inventory and also showing how such an equation can be incorporated in a stand-level decision support system that aims to optimize the forest management for the maximization of wood volume production in southwestern Turkey brutian pine stands.

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