THE INFLUENCE MECHANISM OF A NONLINEAR SYSTEM ON PRECIPITATION INFILTRATION

HE, Y. – WANG, G.[*]

*Institute of Hydrogeology and Environmental Geology, CAGS, 050061 Shijiazhuang, China
(phone: +86-186-3308-5615; fax: +86-311-6759-8531)*

**Corresponding author
e-mail: ihegwh@163.com; phone: +86-157-3696-0130; fax: +86-311-6759-8537*

Abstract. In order to uncover the heavy precipitation percolation feature and formation mechanism intensity in the deep vadose zone called black box, the nonlinear dynamical system theory was introduced in this paper. By in-situ monitoring the water dynamics in Taihang piedmont deep aeration zone, the heavy precipitation infiltration-evaporation process, wetting front advancing process, water storage changing process were analyzed. The result showed that: (1) the deep vadose zone was a "balance-unbalance-balance" nonlinear dynamical system. And the water infiltration was a typical nonlinear process; (2) the forming mechanism comprised of the internal control parameter vector C determined by vadose zone lithology, and the external forcing terms G(t) determined by rainfall, evaporation and human activities ect.; (3) when the time, space scale is large enough, the non-autonomous system could be transformed into a free system and linear system.
Keywords: *deep vadose zone, soil water distribution, heavy precipitation, nonlinear infiltration, TDR100 system*

Introduction

A nonlinear system cannot be described by a linear mathematical model. In the system at least one part or all parts have nonlinear characteristics and have nonlinear interaction among them. The basic characteristics of nonlinear system lead to the diversity and multiple scales of dynamic process in the system, so the nonlinear system's study is more complicated than the linear system's study. The nonlinear system cannot satisfy superposition principle and its mass action is greater than the sum of partial actions. The interaction of many small nonlinear factors in the nonlinear system may generate unpredictable results. The continual dynamic system could be expressed by the following equations (Li, 2006).

$$\begin{cases} \dot{X}_1 = f_1[(x_1 \mathrm{K}\ x_n)(c_1 \mathrm{K}\ c_m)] \\ \dot{X}_2 = f_2[(x_1 \mathrm{K}\ x_n)(c_1 \mathrm{K}\ c_m)] \\ \qquad \mathrm{M} \\ \dot{X}_n = f_n[(x_1 \mathrm{K}\ x_n)(c_1 \mathrm{K}\ c_m)] \\ \qquad \mathrm{M} \end{cases} \qquad (Eq.1)$$

At least one of f1, …, fn is nonlinear function, C = (C1, C2, …, Cm) represents control parameter vector and F = (f1, f2, …, fn). So *Equation 1* can be considered a vector.

$$Z = F(X, C) \tag{Eq.2}$$

The system described by *Equation 2* has no external forcing and is a free system. If external forcing G (t) is introduced the system will be a forcing system which can be expressed by *Equation 3*.

$$Z = F(X, C) + G(t) \tag{Eq.3}$$

Equation 2 contains the time t so the system is known as non-autonomous system which can be expressed by *Equation 4*.

$$Z = F(X, C, t) \tag{Eq.4}$$

In *Equations 3* and *4,* G(t) and t are considered as a new state variable and the system can be conformed to free system.

The continual decline in groundwater level in Taihang Piedmont led to the depth of zone of aeration becoming greater (Baram et al., 2016). This is the typical nonlinear system under the interaction of precipitation, evaporation and human activity.

Every layer in the zone of aeration can be marked as Z_1, Z_2, …, Z_n, f_1, f_2,…,f_n are the nonlinear function of soil water potential or water content. The parameters of soil hydraulic properties can be expressed by C_1, C_2, …, C_m. The external forcing G(t) contains precipitation, evaporation, temperature and mankind's activity. The infiltration process of heavy precipitation takes place on the zone of aeration and be controlled by this nonlinear dynamic system, such as spatial variability (Dahan et al., 2017) inner the system. Meanwhile, the infiltration of heavy precipitation has an adverse effect on this nonlinear system, such as a new external forcing was added to the system over a period of time and cause the infiltration process analysis and simulation of nonlinear dynamic system more complicated (Gupta et al., 2018). Most of the traditional seepage researches in the zone of aeration separate the soil from the infiltration-evaporation process and less likely to carry the above two into a system (Turkeltaub et al., 2016) to explore the inner migration process and mechanism of black box.

In order to subtly study the soil hydraulic characteristics in the zone of aeration under various factors, this paper introduced the nonlinear dynamic systemic mathematic principle and take the heavy precipitation infiltrated into the zone of aeration and the soil as an entity (Min et al., 2017). And this paper analyzes the infiltration-evaporation process, moisture storage variation process, etc and tried to apply the nonlinear dynamic system to interpret the mechanism of the process.

Materials and methods

The study area

From 2012-2013, the study carried out in key field experimental base of the Ministry of Land and Resources in Zhengding, China. The study area has a temperate and semi-humid continental monsoon climate with clearly cut seasons. The annual average temperature is 12.9 °C, the average temperature in July is 26.5 °C and is the highest in a year and in January is the lowest with the temperature is -2.9 °C. The precipitation is focus on June-September and the annual average precipitation is 569.8 mm. The annual average evaporation is 1092.3 mm. The study time period was from April 15th to June

5th. *Figure 1* showed that the heavy precipitation with 70 mm at April 18th, 16 mm at April 21th and 26 mm at April 24th and long-time evaporation process from April 24th to May 16th.
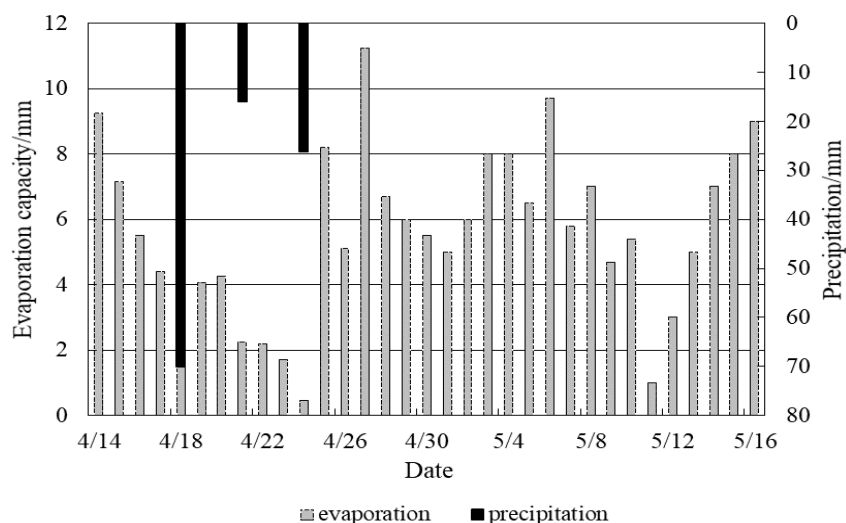

Figure 1. *Rainfall and evaporation (2012)*

Zhengding is located in the Piedmont Clinoplain in the midst of Taihang Mountain with the depth of shallow groundwater depth 36.1-49.1 m. The soil is composed mainly of silt clay and sandy clay, and specific gravity of soil is 2.2-2.74 g/cm³ (Min et al., 2018).

Monitoring system layout

The study uncovered a typical multi-phase monitoring profile with 33 m depth, 2.5 m inner radius and 3.5 m outer diameter. The Time-Domain Reflectometry system developed by CAMPBELL Company in USA was applied to monitor the soil moisture. The detectors were installed on the inside of the typical profile. 36 CS630 were installed to measure the soil volumetric water content, 36 WATERMARK200-253/257 to measure the soil matric potential and 36 MODEL109 to measure the soil temperature. The buried depth of three kinds of detectors was the same. From the depth of 0.1 m, a detector was installed every 20 cm within 2 m and every 30~70 cm below 2 m. All the detectors were calibrated by the neutron probe and soil sampling.

The monitoring period was from August in 2011 and the monitoring interval was 0.5 h. The TDR systemic time was set up according to the Beijing time. At the same time the meteorological observation was carried out and its observation interval was 0.5 h. It includes 19 general meteorological data, such as precipitation, evaporation, the solar radiation, etc.

Results and discussion

The effect of lithology on infiltration

The control parameter vector $C = (C_1, C_2,…, C_m)$ was internal effect factor of the nonlinear dynamic system. For the nonlinear dynamic system in the zone of aeration the

control parameters are determined by the lithology of every layer. Suppose that C1 was soil particle size, C2 was soil specific retention and C3 was hydraulic conductivity of soil, and among them C1 and C2 was a constant at a particular depth and C3 was a variable changing with the soil moisture content. The above three parameters had an effect on the nonlinear function and there had correlation among them. And the above three parameters were determined by the soil lithology characteristics.

The TDR system was applied to measure and analyze the soil matric potential during the three precipitation-infiltration process and evaporation process. Because the evaporation intensity was great during the April to June so the measurement value was the value at 20:00 of the day for the evaporation process was essentially completed. *Figure 2* shows that 12 detectors measure the soil matric potential daily at different depth of the profile.

According to the measured value at different times, the initial matric potential was significantly lower than the adjacent layers. The initial matric potential was measured under the longstanding stable weak-evaporation condition, and so the main influencing factor of the difference of the soil matric potential at different layer was the lithology. Besides, at April 22th, three days behind the precipitation, the matric potential drop rapidly by 20 cm and the falling speed was greater than other layers. This phenomenon illustrated that water retention capability of soil at 30 cm depth was poor and the soil permeability was high so this layer was the best water passage in relation to the other layers. At the 180 m depth there has the similar simulation.
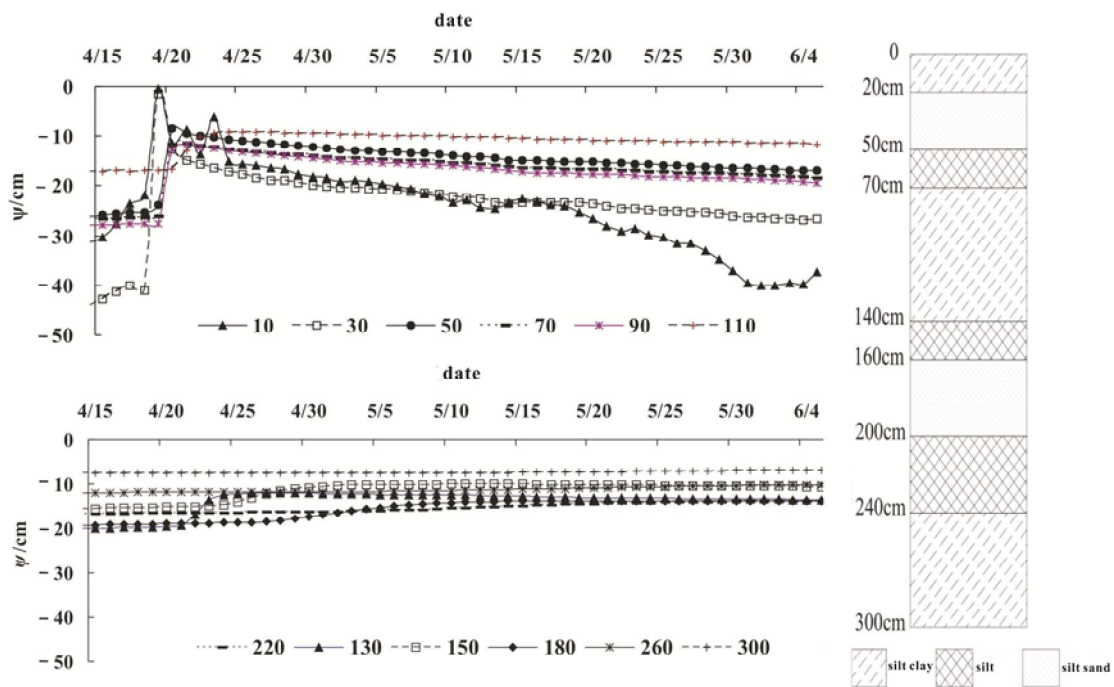

Figure 2. Soil matrix potential variations of different depth

The effect of lithology on soil water storage

The lithology has a significant effect on the soil water storage. The precipitation and evaporation changes the soil water storage and this process is a dynamic changing process. For the water storage compute of zone of aeration under one-dimensional

condition, the soil volumetric water content at different depth can be measured, then accumulated the difference value of the soil water content at different time period and finally get the dynamic soil water storage. The CS630 detectors were applied to monitor the soil water content at different depth before heavy precipitation, 4 h and 720 h after precipitation. *Table 1* showed the soil moisture content dynamics at different time period by calculating changes in moisture content at each layer and then cumulated to get the total storage variation.

Table 1. *Soil moisture content dynamics and storage variations at different depth*

Moisture Depth/cm	$\theta_1(0h)$	$\theta_2(4h)$	$(\theta_2-\theta_1)$	ΔW_1/mm	$\theta_3(720h)$	$(\theta_2-\theta_3)$	ΔW_2/mm	$(\theta_3-\theta_1)$	ΔW_3/mm
10	0.177	0.369	0.192	19.2	0.16	0.209	20.9	-0.017	-1.7
30	0.106	0.26	0.154	30.8	0.164	0.096	19.2	0.058	11.6
50	0.103	0.23	0.127	25.4	0.18	0.05	10	0.077	15.4
70	0.105	0.117	0.012	2.4	0.15	-0.033	-6.6	0.045	9
90	0.123	0.124	0.001	0.2	0.14	-0.016	-3.2	0.017	3.4
110	0.105	0.104	-0.001	-0.2	0.13	-0.026	-5.2	0.025	5
130	0.124	0.124	0	0	0.14	-0.016	-3.2	0.016	3.2
150	0.133	0.134	0.001	0.2	0.16	-0.026	-5.2	0.027	5.4
180	0.12	0.12	0	0	0.15	-0.03	-9	0.03	9
220	0.146	0.145	-0.001	-0.4	0.16	-0.015	-6	0.014	5.6
260	0.25	0.25	0	0	0.265	-0.015	-6	0.015	6
300	0.045	0.045	0	0	0.045	0	0	0	0
Sum				77.6			5.7		71.9

The precipitation was 70 mm and lasted 4 h. θ_1, θ_2, θ_3 represented the soil volumetric water content 0 h, 4 h and 720 h after precipitation respectively, ΔW_1 represented the storage variation 4 h after precipitation at different depth, ΔW_2 represented the storage variation between 4 and 720 h after precipitation at different depth, ΔW_3 represented the storage variation between 0 and 720 h after precipitation at different depth

In *Table 1,* the storage variation was up to 30.8 mm between 0 and 4 h after precipitation at 30 cm depth and was the maximum at the depth of 0-300 cm. that is because there were coarse soil particles and great porosity. The soil water storage can be increased rapidly during heavy precipitation and dropped quickly from 26% to 18% after 24 h. The high infiltration capacity of this layer was proved again. The storage variation simulation at 180 cm depth was similar to the above situation.

The above phenomenon was determined by the difference of the soil lithologic characters. For the high-content of sands at depth of 30 cm and 180 cm so the soil was named as silty sand according to the international standard. If the natural soil was not disturbed, it is difficult to monitor the mid-deep part of the vadose zone directly and accurately, so the hydrological cycle problem was settled from the point of lithology may be a new idea.

The effect of two and evaporation on infiltration and soil water storage

The nonlinear dynamic system was affected by the external forcing G (t) and was transformed to non-autonomous system. Its character was varying with the time significantly. Two precipitations, relative to the first heavy precipitation, and

evaporation were consistent with this character. The shallow vadose zone approached saturation after heavy precipitation, the soil water content of deep- layers was increased and the permeability of unsaturated soil enhanced. So the two precipitations were easily to form the infiltration and increased the soil water storage. For the deep vadose zone two precipitations were hardly leaked to the groundwater level during the short time period. The process of evaporation varying with time was more significantly, especially at different layers. G (t) in the shallow non-autonomous system was nonlinear and below a certain depth was linear. In this paper the certain depth was 220 cm. When evaporation tends to be constant, G (t) was a constant and the system reached a new balance, the non-autonomous system was transformed to free system. The water storage of vadose zone reverted to the state before heavy precipitation and the storage was again determined by the lithology characteristics.

Conclusions

According to the analysis of the process of infiltration-evaporation, wetting front progradation, storage variation and dynamic variation of total water potential, the infiltration process occurring in the nonlinear dynamic system was typical nonlinear process. The formation mechanism of nonlinear infiltration process includes the inner control parameter vector C and the outer external forcing G(t). When the system suffering the heavy precipitation and evaporation becomes a non-autonomous system and the infiltration process will be determined by G(t) varying with time. When the system tends to be constant, the system becomes a free system and the infiltration process will be determined by the inner parameter vector C, such as the lithology of each layer in vadose zone. And further study is required to establish the specific range of soil particle sizes which can determine the lithology characteristics.

REFERENCES

[1] Baram, S., Couvreur, V., Harter, T., Read, M., Brown, P. H., Kandelous, M., Hopmans, J. W. (2016): Estimating nitrate leaching to groundwater from orchards: Comparing crop nitrogen excess, deep vadose zone data-driven estimates, and HYDRUS modeling. – Vadose Zone Journal 15(11): 1-13.

[2] Dahan, O., Katz, I., Avishai, L., Ronen, Z. (2017): Transport and degradation of perchlorate in deep vadose zone: implications from direct observations during bioremediation treatment. – Hydrology and Earth System Sciences 21(8): 4011.

[3] Gupta, P. K., Yadav, B. K. (2018): Spatial and Temporal Nitrate Transport in Deep Heterogeneous Vadose Zone of India's Alluvial Plain. – In: Thangarajan, M. (ed.) Groundwater. Springer, Singapore, pp. 171-178.

[4] He, Y., Lin, W., Wang, G. (2015): The effect of winter flood irrigation with saline water on groundwater in a typical irrigation area. – Water Science and Technology: Water Supply 15(2): 356-361.

[5] Li, S. Y. (2006): Nonlinear Science and Complexity Science. – Harbin Institute of Technology Press, China.

[6]   Min, L., Shen, Y., Pei, H., Jing, B. (2017): Characterising deep vadose zone water movement and solute transport under typical irrigated cropland in the North China Plain. – Hydrological Processes 31(7): 1498-1509.

[7]   Min, L., Shen, Y., Pei, H., Wang, P. (2018): Water movement and solute transport in deep vadose zone under four irrigated agricultural land-use types in the North China Plain. – Journal of Hydrology 559: 510-522.

[8]   Turkeltaub, T., Kurtzman, D., Dahan, O. (2016): Real-time monitoring of nitrate transport in the deep vadose zone under a crop field – implications for groundwater protection. – Hydrology and Earth System Sciences Discussions 20: 3099-3108.