INCIPIENT CONDITION OF SEDIMENT MOTION IN GREAT DIMENSIONLESS FLOW DEPTH

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Abstract. The incipient condition of granular sediments in great dimensionless flow depth was studied. A series of experimental runs were performed with four kinds of sediments in a 28 m long, 6 m wide flume and the data concerning incipient velocity, water depth and sediment particle size were collected to determine the relationship between incipient velocity and dimensionless water depth. The similarity condition between experimental flow and prototype flow with great dimensionless water depth was analysed according to the logarithmic velocity distribution law in the open channel firstly. The analysis results indicate that water depth cannot be used as a directing-variable for the incipient velocity of granular sediments in great dimensionless flow depth. In order to solve this problem, boundary layer momentum thickness was introduced into flume experiment to replace water depth as the directing-variable. Furthermore, an empirical formula for the incipient velocity of granular sediment in great dimensionless flow depth that includes the proportion of fluid γ, the proportion of granular sediment γ_s, the acceleration of gravity g and the dimensionless water depth (H/d) was proposed and the calculated results are more accurate and reasonable compared to previous studies, especially when the dimensionless water depth is greater than 10^4.

Keywords: *incipient velocity, granular sediment, the ratio of water depth to particle size (H/d), similarity condition, boundary layer momentum thickness*

Introduction

Almost all sediment-related problems in water, including water quality and pollution, scouring, deposition, and issues related to construction and management of reservoirs and canals, and wetland restoration, are associated with the incipient condition of sediment motion (Venditti et al., 2006). In a lake, the bottom currents due to external forces (e.g., winds) can suspend the bottom sediments, thus enhancing the release of contaminants and nutrients from the bed into the water column (Zhang et al., 2018; Zhang and Yu, 2017). The incipient velocity of sediment is an important parameter to estimate sediment motion as well as riverbed evolution, and to solve the engineering problems of riverbed deformation, bank protection engineering, channel stability and other engineering problems (Zou et al., 2017; Mao et al., 2011). The study on the incipient condition of sediment motion has also great significance to the protection of marine ecological environment, hence it has been a hot topic for decades.

There are two major parameters to describe the incipient condition of sediment motion, i.e., threshold bed shear stress (Rijn, 1984; Chien and Wan, 1999; Yu and Lim, 2003; Zhang and Yu, 2017) and incipient velocity (Beheshti et al., 2008; Francisco et al., 2014). Incipient velocity is more widely used in practices, because only when the flow velocity is sufficiently great, the sedimentary particles which belong to a flat-bed can be dislodged and start to move (Righetti and Lucarelli, 2007; Luo, 2011). In

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addition, the incipient velocity of sediment is the basic of mechanics of sediment motion and riverbed evolution, and is also an important parameter to solve the sediment problems of riverbed deformation, bank protection engineering, channel stability and other engineering problems (Zou et al., 2017; Mao et al., 2011).

There are a lot of efforts on investigating the incipient condition of sediment motion and many useful formulae have been proposed and successfully applied in practical engineering. As early as 1935, according to the experimental data, Hjulstrom found that when the sediment particle size was 0.2-0.3 mm, the incipient velocity of sediment had a minimum value (Phillips, 1992) and since then, scholars from all over the world carried out many experimental research and theoretical analysis, and derived many formulae for the incipient velocity of sediment from the perspective of mechanics and random process. Xie (1981) recommended several representative former Russia formulae such as Shyamov formula (*Eq. 1*), which can be used to calculate the incipient velocity of the granular sediments with the particle size $d > 0.15$ mm and Goncharov formula (*Eq. 2*).

$$v_c = 1.14\sqrt{\frac{\rho_s - \rho}{\rho} gd}\,(\frac{h}{d})^{\frac{1}{6}} \qquad \text{(Eq.1)}$$

$$v_c = 1.07\sqrt{\frac{\rho_s - \rho}{\rho} gd}\,\lg(\frac{h}{d})^{\frac{1}{6}} \qquad \text{(Eq.2)}$$

where d = the particle size, v_c = the incipient velocity of sediments, h = the depth of water, ρ and ρ_s were the density of fluid and sediment particle respectively, g = acceleration of gravity, ξ and m were the undetermined coefficients. Considering the viscosity of fine sediment, many Chinese scholars have proposed new formulae for the incipient velocity of sediment. Dou (1960) proposed a formula for the incipient velocity of sediment in shallow water in 1960 and modified it in 1974, such as *Equation 3*.

$$v_c = 0.32(\ln 11\frac{h}{k_s})[(\frac{\rho_s - \rho}{\rho})gd + 0.19(\frac{\varepsilon_k + dh\delta}{d})]^{0.5} \qquad \text{(Eq.3)}$$

where $\delta = 0.213 \times 10^4$ cm; $\varepsilon_k = 2.56$ cm^2/s^2, k_s is riverbed roughness, and for the flat bed, $k_s = 0.5$ mm when $d \le 0.5$ mm and $k_s = d$ when $d > 0.5$ mm. Zhang (1961) proposed *Equation 4*, which is the formula of Wuhan Institute of Hydraulic and Electric Engineering and adopted in Chinese code.

$$v_c = [1.76(\frac{\rho_s - \rho}{\rho})d + 0.000000605(\frac{10+h}{d^{0.72}})^{0.5}(\frac{h}{d})^{0.14} \qquad \text{(Eq.4)}$$

However, the parameters introduced by previous research results which are mostly determined by the flume experiments would affect the accuracy of the formulae, because the water depth in flume is always limited to 0.1-0.5 m, whereas the actual estuary water depth can reach 5-10 m, sometimes even about 20 m (Zeng et al., 2010). Compared with other formulas, the formula (*Eq. 5*) of Sha (1965) payed more attention

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to the influence of increased water depth on the incipient velocity of sediment, which was closer to the natural river (Zhang, 2012).

$$v_c = \sqrt{\frac{\rho_s - \rho}{\rho} gd} h^{1/5} [266(\frac{\delta}{d})^{1/4} + 0.66 \times 10^9 (0.7 - \varepsilon)^4 (\frac{\delta}{d})^2]^{1/2} \quad \text{(Eq.5)}$$

where δ = the film water thickness and $\delta = 0.0001\ mm$; ε was the porosity and $\varepsilon \cong 0.4$). Whereas, Han et al. (1982) considered that the formula of Sha (1965) took the influence of the bonding force caused by contact between sediment particles and the porosity on the incipient velocity of sediment into account, but the mechanical mechanism was not described and expressed clearly. Therefore, the formula for the incipient velocity of sediment needs to be further studied.

The formulae of the incipient velocity of sediment are based on limited water depth which may result in their inapplicability (Zeng et al., 2010). The formula form $v_c = kd^{1/3}h^{1/6}$ is almost adopted to calculate the incipient velocity of non-viscous sediment (Karmer, 1935; He et al., 2002). Nie et al. (2004) summarized dozes of the existing formulas and derived the unified formula (*Eq. 6*), whereas the values of ξ and m were slightly different in different formulae.

$$v_c = \xi \sqrt{\frac{\rho_s - \rho}{\rho} gd} (\frac{h}{d})^m \quad \text{(Eq.6)}$$

In most cases, the value of *m* is 1/6 and the of value ξ is usually between 3. 37 and 7 (Nie, 2004). He et al. (2002) believed that ξ was not a constant, but within a certain range and changed with the relative exposure of particles on the bed surface. However, it is found that the value of ξ at different flow levels is variable according to the analysis of the 246 groups of data on sediment initial motion at Yichang station which is one of the reservoirs of Three Gorges in China (Li et al., 2006). Li et al. (2006) used the normative formula of Wuhan Institute of Hydraulic and Electric Engineering and Shyamov formulae to calculate the incipient velocities of sediment based on the different flow levels at the Yichang station, and there was a huge error between the calculated results and the field measured data, especially in the case of huge flow. In addition, the coefficient ξ of *Equation 6* varies greatly when it is used to calculate the incipient velocities of the granular sediments with different particle sizes in the flow with the same water depth, i.e., Lu (1991) proposed the formulae for the incipient velocity of the pebble in the Yangtze river (its *H/d* range is 80 ~ 1000) and sandy and muddy (the range *H/d* of was 10000 ~ 100000), which based on the field measured data of sediment initial motion in the Yangtze river, but the value of ξ are 0.95 and 1.47, respectively, between which the difference is 1.55 times. Therefore, it is still necessary to study the incipient velocity of sediment under the premise of fully considering the influence of water depth or relative water depth on the incipient velocity of sediment.

To ensure that the formulae for the incipient velocity of sediment obtained by the flume experiments are more accurate and reliable, the similarity conditions between the experimental flow and the prototype flow with great dimensionless water depth should be satisfied (Ferro, 1999; Falcão et al., 2014; Dornbrack and Schumann, 1993; Steward and Tennankore, 1977). According to the force analysis of the sediment particles on the horizontal bed without seepage, only the drag force and the lifting force are favorable to

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the initial motion of sediment particles, except the geometric status of sediment particles and the bed roughness (Niven, 2010; Bohorquez and Fernandez-Feria, 2008; Mao et al., 2011). Therefore, the distribution of the flow velocity on vertical line in the experimental flow and the prototype flow with great dimensionless water depth should satisfy the principle of similitude to ensure the accuracy of the experimental results, because the distribution of flow velocity would be affected by different water depth, which would have an impact on the Kinematic similarity between the experiment flow and the prototype flow with great dimensionless water depth. The velocity distribution in experimental flow and natural open channel flow generally adopt the logarithmic formula of velocity distribution (Yu and Tan, 2006; Meftah and Mossa, 2016) and the formula can be written as

$$\frac{u}{u_*} = \frac{1}{k}\ln(\frac{y}{\Delta}) \qquad \text{(Eq.7)}$$

where y = the height from the bed, u = the velocity at the height y, u_* = the friction velocity, κ = the Carmen coefficient, Δ = the rigidity of bed sediment and $\Delta = K_s/30.2$ (Nikuradse, 1933), K_s is the characteristic particle size of the bed surface. Hence, the similarity conditions between the experimental flow and the prototype flow with great dimensionless water depth would be analyzed firstly according to the logarithmic velocity distribution law in open channel.

In this study, a series of experimental runs were conducted and the previous data were collected. Finally, a new formula for the incipient condition of the granular sediment in great dimensionless water depth was proposed. Section 2 described the analysis of Similarity condition between experimental flow and prototype flow with great dimensionless water depth. Section 3 described the derivation and the structure of the empirical formula for the incipient velocity of granular sediment in the flow with great dimensionless water depth. Section 4 described the test materials, experimental setup, and test procedure. In section 5, the experimental results were presented, and the parameters of the empirical formula were determined. In addition, the soundness and the accuracy of this empirical formula were verified by comparing with previous studies. Finally, section 6 presented the main conclusions of the study and the recommendations for future studies.

Theoretical basis

Similarity requirement for sediment motion in different flow depths

In this paper, the similarity condition between the experimental flow and the prototype flow with great dimensionless water depth was analysed based on the logarithmic velocity distribution law in open channel. It is assumed that the experimental flow was to be able to fully simulate the prototype flow with great dimensionless water depth. Some assumptions are made to better illustrate the similarity between the experimental flow and the prototype flow. The flow distributions in an experimental flume and in a prototype flow are shown in *Figure 1*, where the subscript m denotes model (experimental flume) and the subscript p denotes the prototype. H_p and H_m is the flow depth in the experimental flume and in the prototype flow, respectively. Suppose $H_p/Hm = \lambda$. The velocity in the experimental flow at the height y_{m1} and y_{m2}

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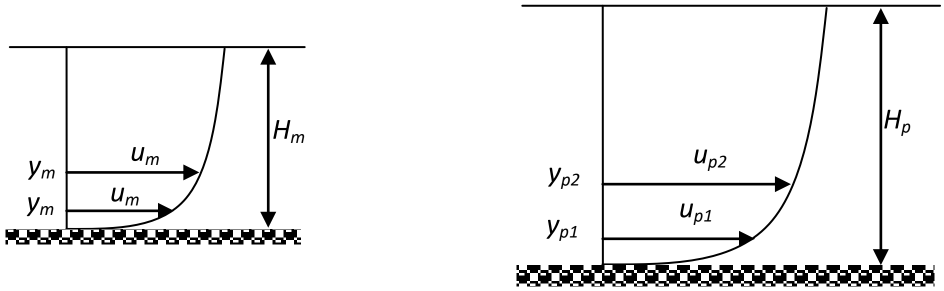
from the bed surface are assumed as u_{m1} and u_{m2}, and the corresponding velocities in the prototype flow with great dimensionless water depth at the height $y_{p1} = \lambda y_{m1}$ and $y_{p2} = \lambda y_{m2}$ from the bed surface are assumed as u_{p1} and u_{p2}. The particle sizes of the experiment flume bed and corresponding prototype bed are assumed as d_m and d_p, respectively.



(a) Experimental flow (b) Prototype flow

Figure 1. *Vertical velocity distribution*

Suppose $K_s = d$ (Bennett and Best, 1995; Mianaei et al., 2010) for the uniform sediment. Hence,

$$\frac{u_{m1}}{u_{*m}} = \frac{1}{\kappa}\ln(30.2\frac{y_{m1}}{d_m}) \quad \text{(Eq.8)}$$

$$\frac{u_{m2}}{u_{*m}} = \frac{1}{\kappa}\ln(30.2\frac{y_{m2}}{d_m}) \quad \text{(Eq.9)}$$

$$\frac{u_{p1}}{u_{*m}} = \frac{1}{\kappa}\ln(30.2\frac{y_{p1}}{d_m}) \quad \text{(Eq.10)}$$

$$\frac{u_{p2}}{u_{*m}} = \frac{1}{\kappa}\ln(30.2\frac{y_{p2}}{d_m}) \quad \text{(Eq.11)}$$

Equation 8 is divided by *Equation 9*, and the parameters u_{*m} can be eliminated. In a similar way, *Equation 11* is divided by *Equation 10*, and the parameters u_{*p} can be also eliminated. Hence,

$$\frac{u_{m1}}{u_{m2}} - \frac{u_{p1}}{u_{p2}} = \frac{\ln(30.2\frac{y_{m1}}{dm})}{\ln(30.2\frac{y_{m2}}{d_m})} - \frac{\ln(30.2\frac{y_{p1}}{d_p})}{\ln(30.2\frac{y_{p2}}{d_p})} \quad \text{(Eq.12)}$$

Equation 12 can be further simplified, and the similarity condition between the experimental flow and the prototype flow with great dimensionless water depth can be written as

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$$\frac{u_{m1}}{u_{m2}} - \frac{u_{p1}}{u_{p2}} = \frac{\ln(\frac{y_{m2}}{y_{m1}})[\ln(\frac{d_p}{d_m}) - \ln(\lambda)]}{\ln(30.2\frac{y_{m2}}{d_m})\ln(30.2\frac{y_{p2}}{d_p})} \quad \text{(Eq.13)}$$

According to kinematic similarity criteria, it can be seen from *Equation 13*, when $\lambda = d_p/d_m$, u_{m1}/u_{m2} is equal to u_{p1}/u_{p2}. In other words, only when the dimensionless water depth (the ratio of water depth to particle size, H_m/d_m) in experimental flow is the same as that in corresponding prototype flow H_p/d_p, the flow velocity near the particles in experimental flow is similar with that in corresponding prototype flow and the motion of the sediment is similar in both scenarios. Otherwise, if $\lambda < d_p/d_m$, u_{m1}/u_{m2} is larger than u_{p1}/u_{p2} which indicates that the flow velocity near the sediment particles in experimental flow is higher than that in corresponding prototype flow, and the sediment particles in experimental flow would be easier to be dislodged than that in corresponding prototype flow. On the other hand, when $\lambda > d_p/d_m$, u_{m1}/u_{m2} is smaller than u_{p1}/u_{p2}, which indicates that the flow velocity near the sediment particles in experimental flow would be lower than that in corresponding prototype flow, and the sediment particles in experimental flow are then more difficult to be dislodged than that in corresponding prototype flow. It is seen that similarity condition between experimental flow and corresponding prototype flow is that the sediment particle size scale (d_p/d_m) equal to water depth scale (H_p/H_m), that is, $H_p/d_p = H_m/d_m$. Hence, the similarity between experimental flow and corresponding prototype flow is necessary to ensure the similar flow velocity near the sediment particles near bed in experimental flow and corresponding prototype flow.

In line with the similarity analysis between the experimental flow and the prototype flow in great dimensionless water depth, the current formulae deduced from flume experiments may be not suitable for calculating the incipient velocity of the granular sediment motion in great dimensionless depth of water, which requires a wide range of H/d in the flume experiment, for example, the water depth varies from 10 to 40 m, and the sediment particle sizes are between 0.2 and 2 mm in the middle reaches of the Yangtze river (Lu, 1991), thus the range of H/d is more than 200,000, and, the H/d range from 2 to 2030 according to the 272 groups of experimental data collected by author. On the other hand, non-natural sediments such as the homogeneous model sediments are always used in flume experiments (Geiger and Durnford, 2000; Thomas and Calantoni, 2001; Jain and Juans, 2009), and the particle size of the homogeneous model sediment is relatively larger than that of the natural sediments, which cannot be used to simulate the motion of cohesive sediments. Because the particle size of the cohesive sediments is always less than 0.06 mm (Grabowski et al., 2011; Jean Berlamont et al., 1993), and the viscosity between cohesive sediment particles will affect the incipient condition of sediments (Mehta, 1984 and1989; Lumborg and Windelin, 2003). Therefore, the limitation of experimental flume size would affect the accuracy of the existing formulas for the incipient velocity for the granular sediments in the flow with great dimensionless water depth, and and the water depth in flume experiment should not be used as the direct variable of the formulas for the incipient condition of the sediment in the flow with great dimensionless water depth.

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Boundary layer momentum thickness calculation

In this paper, boundary layer momentum thickness was introduced into the flume experiment as the independent variable to calculate the incipient velocity of the granular sediment in the flow with great dimensionless water depth. Boundary layer thickness reflects the block effect of bed wall and Roux (2010) regarded that the incipient motion of sediment is mainly affected by the flow near bed, that is, the flow near the boundary layer and the influence of the flow velocity outside the boundary layer on the sediment motion could be ignored in deep water area. Due to the viscous effect of fluid, the change of flow velocity in boundary layer will cause the loss of momentum in the boundary layer. The loss of momentum in the boundary layer can be explained by the momentum thickness of the boundary layer. Therefore, the boundary layer momentum thickness can be taken as a direct factor affecting sediment motion, which should be used to calculate the incipient velocity of sediment, rather than the water depth.

In general, boundary layer momentum thickness δ^1 is used to describe the boundary layer thickness intuitively (Hokenson, 1977), which can be written as follows:

$$\rho\delta^1 U^2 = \int_0^{\delta^*} \rho u(U-u)dy \qquad \text{(Eq.14)}$$

where ρ is the density of water, *kg/m*^{3}; δ^1 is boundary layer momentum thickness; δ^* is the boundary layer thickness, *m*. *U* and *u* are supposed to comply with the logarithmic velocity distribution law (Jonsson, 1966). Therefore, *Equation 14* can be written as

$$\rho\delta^1 = \frac{\delta^*}{\ln(30.2\frac{\delta^*}{d})} - \delta^* - \frac{1}{[\ln(30.2\frac{\delta^*}{d})]^2}\int_0^{\delta^*} \ln^2(30.2\frac{y}{d})dy \qquad \text{(Eq.15)}$$

where *y* is the water depth where the velocity is *u*, *m/s*. It is assumed that

$$x = 30.2\frac{y}{d} \qquad \text{(Eq.16)}$$

So *Equation 15* can be derived as *Equation 17*.

$$\delta^1 = \frac{\delta^*}{\ln(30.2\frac{\delta^*}{d})} + 2\frac{\delta^*}{\ln^2(30.2\frac{\delta^*}{d})} \qquad \text{(Eq.17)}$$

In the flume scale, the boundary layer thickness is determined by the water depth of the flume, that is $\delta^* = H$, and the boundary layer momentum thickness in shallow water area such as in the flume can be calculated by *Equation 18*.

$$\delta^1 = \frac{H}{\ln(30.2\frac{H}{d})} + 2\frac{H}{\ln^2(30.2\frac{H}{d})} \qquad \text{(Eq.18)}$$

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Formula form derivation

In this section the formula form for the incipient velocity of the granular sediment in the flow with great dimensionless water depth was analysed. The incipient motion of sediment is mainly affected by incipient condition which should conclude the characteristics of sediment particles, flow conditions and the relationship between flow conditions and sediment particles (Wang et al., 2008). Hence, the incipient velocity of granular sediment is influenced by the proportion of sediment particle, the proportion of fluid, kinematic viscosity coefficient, salinity, acceleration of gravity besides boundary layer momentum thickness. While the kinematic viscosity coefficient and saltiness can be ignored, because the temperature has little impact on the kinematic viscosity coefficient of fresh water and sea water (Balucani et al., 1996). Therefore, the formula form of the incipient velocity for the granular sediment particle can be written as

$$v_c = f(g, \gamma, \gamma_s, d, \delta^1) \qquad \text{(Eq.19)}$$

where γ is the proportion of water, kg/m^3; γ_s is the proportion of sediment, kg/m^3. As mentioned above, *Equation 6* is almost adopted as the formula form to calculate the incipient velocity of sediment, therefore the formula form of the granular sediment in the flow with great dimensionless water depth can be expressed as:

$$v_c = \xi\left(\frac{\gamma_s - \gamma}{\gamma} gd\right)^a d^b \delta^{1c} \qquad \text{(Eq.20)}$$

where ξ, a, b, c are constants. In this paper, $d = D_{50}$. It can be deduced that $a = 0.5$, $b = c$ by the dimensional analysis of *Equation 20*. Therefore, the formulary structure of incipient velocity for the granular sediment particle can be written as

$$v_c = \xi\left(\frac{\gamma_s - \gamma}{\gamma} gd\right)^a d^{0.5}\left(\frac{\delta^1}{d}\right)^c \qquad \text{(Eq.21)}$$

Compared with *Equation 6*, water depth H is replaced by the boundary layer momentum thickness δ^1 in *Equation 21*, which is confirmed to the similarity requirement between the experimental flow and the prototype flow with great dimensionless water depth for sediment initial motion.

Materials and method

Test materials

Four kinds of sediment particles with different particle sizes were used for the tests, which concluded one kinds of model sediments (MS) and three kinds of natural sediments. The homogeneous granular plastic sands were used as the test sediments which can be applied to the model experiment with different scale. The particle size of the homogeneous granular plastic sands in this experiment was 3.2 mm and the proportion of the homogeneous granular plastic sands was 1050 kg/m^3. The natural sediments were from different places: coastal line of Fengxian (FX), Shanghai, China; Huangpu River (HPJ) of Shanghai; coastal line of Zhuhai (ZH), Guangzhou province.

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The natural sediments were dried for 24 h in an oven at a temperature of 105°. After that, the dried sediments were sieved through a series of standard sieves with different diameters of holes to screen suitable sediment particle sizes and eliminate impurities such as small stones and shellfish. The determination range of standard sieves in laboratory is 0.038~6 mm. All the grain size and grain density are summarized in *Table 1*.

Table 1. *Grain size and grain density*

Location of sediment	FX	HPJ	ZH
Grain density (g/cm^3)	2.02	2.23	1.98
Median particle size (mm)	0.33	0.84	1.22
Standard deviation of grain size (μm)	1.6	2.6	0.9
Sorting coefficient	1.85	2.11	1.80

In addition, 272 groups of previous data concerning the incipient velocity of granular sediment, water depth and sediment particle size were collected to verify and fit the formula for the incipient velocity for granular sediments in the flow with great dimensionless water depth. The rang of the sediment proportion of the collected data is from 2010 to 2650 kg/m^3, and the rang of the sediment particle sizes of the collected data are from 0.04 to 22.2 mm. The maximum depth selected from the collected data is 0.47 m.

Experiment setup

The flume experiment was performed in the laboratory to examine the incipient velocities of the sediments with different particle size. The layout of the experimental apparatus is shown in *Figure 2*. The experimental flime was a water-circulating rectangle flume with a length of 10 m, width of 1 m, height of 1 m. A rectangular groove with dimensions of 50 cm length, 1 m width, and 5 cm depth was arranged at the bottom of the rectangular flume. The sample box with different sediment particles was placed in a groove during the experiment. The upper surface of the sample box was aligned with the inner wall of the flume to ensure that the inner surface of the flume was flat. Windows are provided on both sides of the flume to facilitate the observation of sediment motion. Two flow straighteners were installed at the upstream and downstream ends of the flume, respectively, to ensure that the water flowed evenly in the test range of the flume. The water inlet and water outlet are on the left side of the flume which can be controlled by the valve. The valves of water inlet and water outlet were closed during the experiment. A pump was arranged under the water-circulating rectangle flume and connected with the flume, flow meter and flow control valve by the pipe. The pump is ISG type vertical pipe centrifugal pump. The lift of this pump is 60 m and the flow is 45 m^3/h. The flow control valve was used to control the flow in the flume. The flow meter is an electromagnetic flowmeter whose nominal diameter is DN6-DN2000 with wide coverage. The flow velocity in the flume was measured by UDV 3000.

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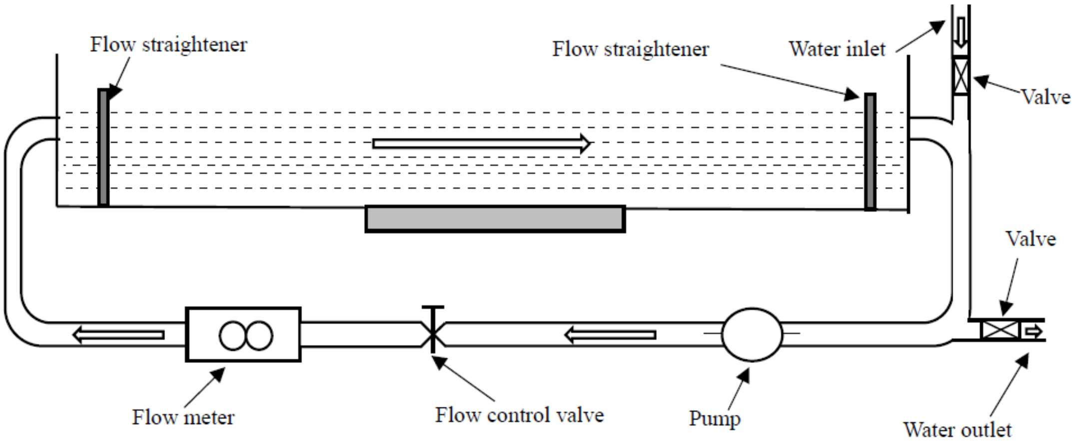

Figure 2. *Experiment setup*

Test procedure

First, tap water was added into the flume through the water inlet on the left side of the flume, and the valve of water outlet was closed at this time. When the water depth reaches a certain level, the valve of water inlet was closed. The depth of the water by artificial control was between 10 and 50 cm. The sample box with the sediment samples inside was then lowered into the square box. Subsequently, the pump was turned on and the flow control valve was then opened slowly. The incipient motion of sediment was observed through the windows on both sides of the flumes. The flow velocity was accelerated very slowly by 1–2 cm/s every minute until a few sediment particles were dislodged from the bed surface, and the flow velocity was recorded at this time. The incipient criterion of sediment is that about 1‰ of the particles dislodged from the bed surface (Kramer, 1935; Chien and Wan, 1983), which also applies to model sediment (Huang et al., 2012). This phenomenon that the sediment particles dislodge from the bed surface continuously took place in both time and space, that is, the sediment motion could always be observed throughout the experiment (Zhang et al., 2017). Because the incipient criterion of sediment processed a certain amount of subjectivity and judgment (He et al., 2003; Huang et al., 2012), there must be some divergent data in the experimental results. In order to minimize the errors of inevitable subjectivity associated with incipient criterion of sediment, the procedure for measuring the flow velocity when the phenomenon that a few sediment particles were dislodged from the bed surface was observed was repeated three times for every sediment samples. Once the incipient criterion of sediment samples was reached, the flow velocity, and the difference of water head were recorded.

Results and discussion

Empirical formula for the incipient velocity for granular sediment in the flow with great dimensionless water depth

In this section, the coefficients ξ and c of *Equation 21* are discussed by means of data fitting and collection. A series of previous data were collected, which include the experimental date of incipient condition of sediment collect by Yang et al. (2006) and a part of the sediment experimental data collected by Brownlie (1981). The basis of the

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data screened from the data collected by Brownlie (1981) is that the suspended sediment concentration range of the collected data should be between 0 PPM and 3 PPM, according to the analysis of sediment transport and the distribution of suspended sediment concentration in the fluid when the sediment particles begin to suspend (Ted et al., 1974) and these screened data included the experiment data of Simons, Bishop and Richardson (1965), Yalin and Karahan (1979), Govt. of West Bengal (1965), Casey (1935), Costello (1974), Davies (1971), Guy at al. (1966), Ho (1939), Mavis et al. (1937), Mutter (1971), Nordin (1976), Oerien (1935), Paintal (1971), Pratt (1970), Taylor (1971). The experimental results are shown in *Table 2*. The collected data are shown in the *Appendix*.

Table 2. *Experimental results*

***H* (*m*)**	**Location of sediment**							
	FX		**HPJ**		**ZH**		**MS**	
	H/d	**V_c (*m/s*)**	***H/d***	**V_c (*m/s*)**	***H/d***	**V_c (*m/s*)**	***H/d***	**V_c (*m/s*)**
0.131	393.94	0.1876	154.76	0.281	106.56	0.2713	40.63	0.0888
0.149	451.52	0.1919	177.38	0.288	122.13	0.2775	46.56	0.0909
0.169	512.12	0.1959	201.19	0.294	138.52	0.2833	52.81	0.0928
0.206	624.24	0.2025	245.24	0.304	168.85	0.2927	64.38	0.0958
0.237	718.18	0.2072	282.14	0.311	194.26	0.2996	74.06	0.0981
0.296	895.45	0.2150	351.79	0.322	242.21	0.3108	92.34	0.1017
0.333	1010.61	0.2194	397.02	0.329	273.36	0.3171	104.22	0.1038
0.366	1109.09	0.2228	435.71	0.334	300	0.3219	114.38	0.1054
0.385	1166.67	0.2247	458.33	0.337	315.57	0.3247	120.31	0.1063
0.426	1290.91	0.2285	507.14	0.343	349.18	0.3302	133.13	0.1081

The coefficients of *Equation 21* were determined respectively using the test results and the collected data with the help of Origin 9.0 and Excel 2017. Firstly, the formal deformation of *Equation 21* was carried out in order to simplify the calculation as follows: the opposite sides of *Equation 21* was taken logarithm respectively, and then the multivariate regression method was used to determine these coefficients. These coefficients were evaluated as follows: $\zeta = 1.74$, $c = 0.18$. The value of $R^2 = 86\%$ indicates that the fit of the obtained model is fairly-good. Hence, the incipient velocity for the granular sediments in the flow with great dimensionless water depth can be expressed as follows:

$$v_c = 1.74\left(\frac{\gamma_s - \gamma}{\gamma} gd\right)^{0.5} \left(\frac{H}{d}\right)^{0.18} \left[\frac{1}{\ln(30.2\frac{H}{d})} + \frac{2}{\ln^2(30.2\frac{H}{d})}\right]^{0.18} \tag{Eq.22}$$

As show in *Figure 3*, the curves plotted using the calculated values based on *Equation 22* are in accordance with the experimental data and the collected data, and this formula possesses an acceptable accuracy for calculating V_c, as 86% of the datasets are within a confidence interval with a relative error of 20%.

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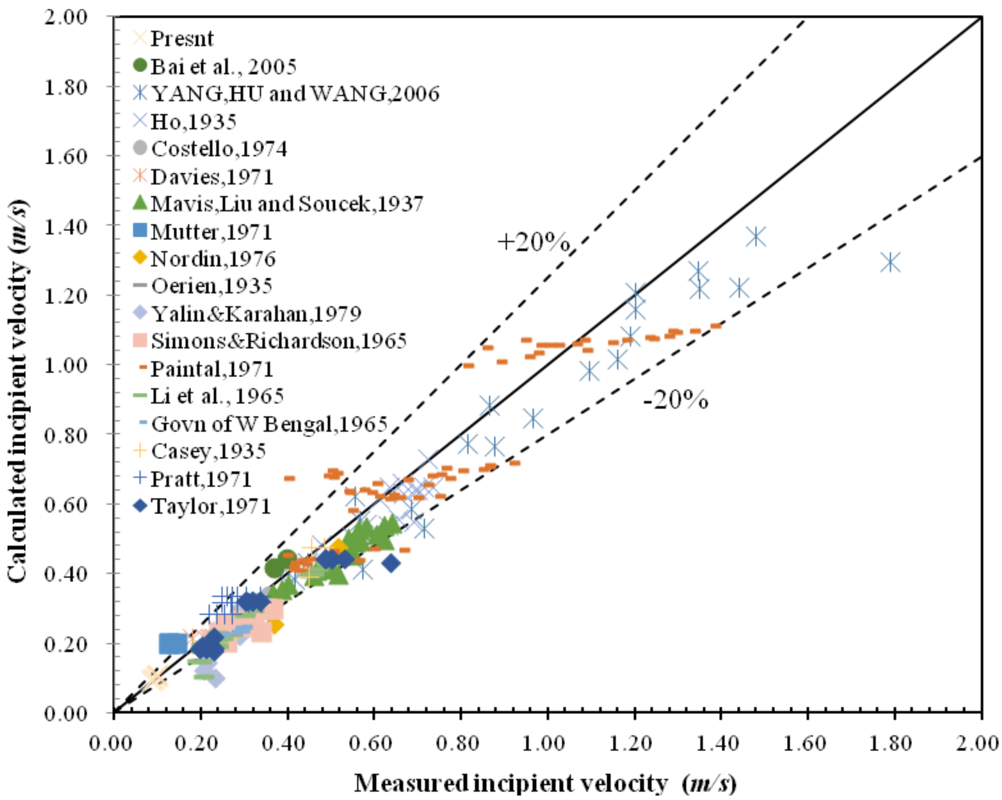

Figure 3. *Comparison between the measured and the calculated incipient velocity*

The incipient velocity of granular sediments in the flow with great dimensionless water depth are plotted according to *Equation 22*, and an incipient velocity diagram is drawn, as shown in *Figure 4*. The incipient velocity of granular sediments in the flow with great dimensionless water depth are plotted as a function of the dimensionless water depth (*H/d*) with the particle size as an independent parameter. It can be seen from *Figure 4* that the incipient velocity of the granular sediments in the flow with great dimensionless water depth increases with the increase of the dimensionless water depth (*H/d*) and the particle size, i.e., the incipient velocity of the granular sediments in the flow with great dimensionless water depth increases from 0.2 m/s to 0.45 m/s when the particle size increases from 0.1 mm to 5 m/s and the great dimensionless water depth is 5000; the incipient velocity of the granular sediments in the flow with great dimensionless water depth increases from 0.2 m/s to 0.625 m/s when the great dimensionless water depth (*H/d*) increases from 5000 to 4000000 and the particle size is 0.5 mm.

Soundness verification this empirical formula

The incipient velocity of granular sediments in the flow with great dimensionless water depth calculated by *Equation 22* proposed in this paper was compared with that calculated by *Equation 4* of Wuhan Institute of Hydraulic and Electric Engineering which has been used as a normative formula for the calculation of incipient velocity of sediment to verify the soundness of *Equation 22*, *Figure 5* depicts the incipient velocities of sediment in the flow with great dimensionless water depth (*H/d*) with various particle size, which are calculated by *Equation 22* and the formula of Wuhan

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Institute of Hydraulic and Electric Engineering. The comparison between calculated results of the empirical *Equation 22* and the normative formula of Wuhan Institute of Hydraulic and Electric Engineering clearly indicates the following:

The incipient velocities of the granular sediments calculated by the two formulas both increases with the increase of the dimensionless water depth (*H/d*) and the particle size of granular sediment.

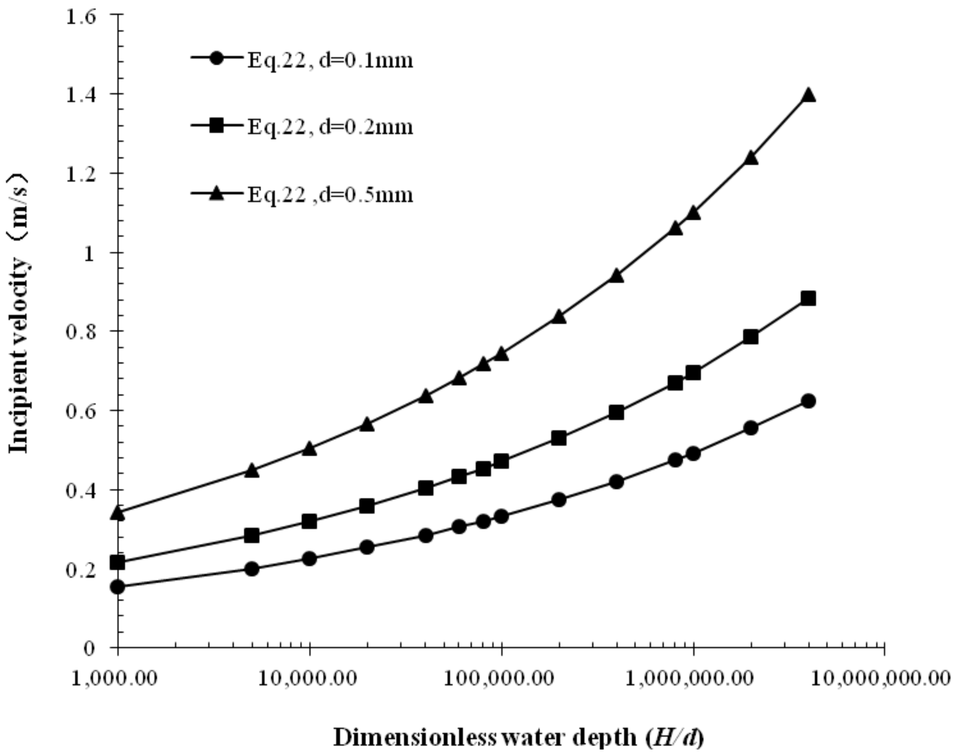

Figure 4. *Incipient velocity diagram*

For $H/d \leq 1 \times 10^4$, the increasing trend of the incipient velocities of the granular sediments calculated by *Equation 16* with the dimensionless water depth (*H/d*) is similar with that calculated by the normative formula of Wuhan Institute of Hydraulic and Electric Engineering, and all increase slowly. Whereas, the incipient velocities of the granular sediments calculated by *Equation 16* is lower than that calculated by the normative formula of Wuhan Institute of Hydraulic and Electric Engineering when the particle size is less than or equal to 0.2 mm. When the particle size is more than 0.2 mm, the incipient velocities of the granular sediments calculated by *Equation 22* is approximately equal that calculated by the normative formula of Wuhan Institute of Hydraulic and Electric Engineering.

For $H/d > 1 \times 10^4$, the increasing trend of the incipient velocities of the granular sediments calculated by *Equation 22* is much slower than that by the normative formula of Wuhan Institute of Hydraulic and Electric Engineering, which demonstrates that there will be a large difference between incipient velocities of granular sediment calculated by *Equation 22* and the normative formula of Wuhan Institute of Hydraulic and Electric Engineering when the dimensionless water depth (*H/d*) is more than 10^4, and the difference increases with the increase of *H/d*. In order to illuminate the

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appearance, Wanxian site is taken as an example, which is one of the reservoirs of Three Gorges in China. The particle size of the sediment at Wanxian site is less than 0.16 mm and the depth of water will reach 100 m when the reservoir is full of water (Wang et al., 2010). But the incipient velocities of the sediment at Wanxian site calculated by *Equation 22* and the normative formula of Wuhan Institute of Hydraulic and Electric Engineering are 1.306 m/s and 0.601 m/s respectively, which differ widely.

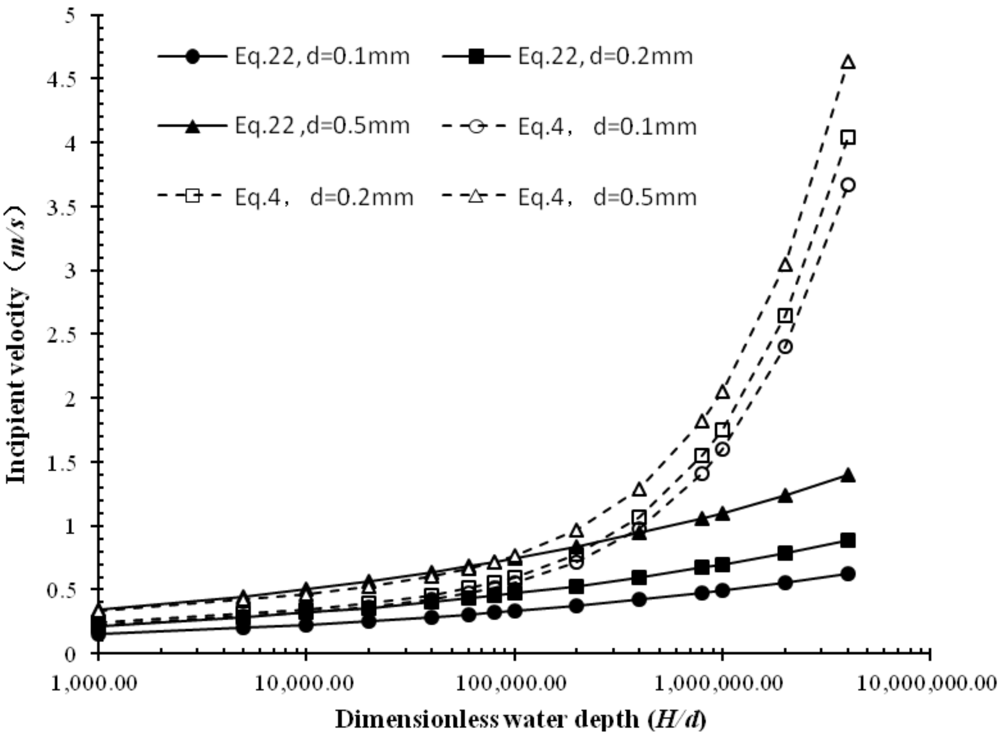

Figure 5. *Comparison between calculated results of the empirical Equation 22 and the normative formula of Wuhan Institute of Hydraulic and Electric Engineering*

Although there is no direct data to determine the rationality of the calculation results *Equation 22* and the formula of Wuhan Institute of Hydraulic and Electric Engineering, it can be concluded that the incipient velocity of granular sediment in the flow with great dimensionless water depth calculated by the formula of Wuhan Institute of Hydraulic and Electric Engineering is certainly higher than the actual situation according to the data of the physical characteristics and environment parameters of the shelf dunes (Zhuang et al., 2004). The area where the depth of water is from 132 to 162 m at continental shelf of the east china sea collected by Zhuang et al. (2004) is taken as the example to verify this claim and there are sand waves with a wavelength of 5 ~ 25 m and a wave height of 0.5 ~ 2 m. In addition, the sediment particle sizes are between 0.125 and 0.188 mm and the local maximum flow velocity is between 0.4 and 0.9 m/s in this area. According to the formula of Wuhan Institute of Hydraulic and Electric Engineering, the incipient velocity of sediment is between 1.43 and 1.91 m/s in this area, while the incipient velocity of sediment calculated by *Equation 22* is between 0.58 and 0.687 m/s. Obviously, the incipient velocity of sediment calculated by the formula of Wuhan Institute of Hydraulic and Electric Engineering is higher than the local maximum flow velocity, which indicates that the sediment particles cannot suspended

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and there are no sand waves. It follows that the calculation results of *Equation 22* are more reasonable in this situation, because the local maximum flow velocity can reach the incipient condition of sediment in this area according to *Equation 22*. According to the above analysis, *Equation 22* proposed in this paper is more credible than the normative formula of Wuhan Institute of Hydraulic and Electric Engineering when they are used to calculate the incipient velocity of granular sediment in the flow with great dimensionless water depth.

Accuracy verification of this empirical formula

In the study, the calculation results of the empirical formula were compared with the field measurement data collected by Brownlie (1981) to verify the accuracy of this empirical formula proposed in this paper. Firstly, the field measurement data collected from the extant studies which included Atchafalaya River Data of Toffaleti (1968), South American River & Canal Data of NEDECO (1973), Red River Data of Toffaleti (1968), and Rio Grande River Data of Nordin and Beverage (1965), in which the dimensionless water depth (*H/d*) was less than 10^4. The calculated results and the collected field measurement data were listed in *Table 3*.

Table 3. *Comparisons with extant experimental data (1)*

Rate of flow (m^3/s)	River width (*m*)	Water depth (*m*)	*d* (*mm*)	Sediment concentration (*ppm*)	Extant data (*m/s*)	Calculated velocity (*m/s*)	Data sources
1073176	313.94	6.889	0.089	4.31	0.396	0.416	Atchafalaya River Data of Toffaleti (1968)
843816	310.89	6.431	0.106	2.80	0.322	0.331	
719226	307.85	6.248	0.096	8.95	0.374	0.319	
637110	307.85	6.218	0.137	5.60	0.333	0.359	
51000	93	1.8	0.12	2.92	0.305	0.282	South American River & Canal Data of NEDECO (1973)
206707	159.41	3.505	0.103	7.88	0.370	0.297	Red River Data of Toffaleti (1968)
2859.9	25.603	0.369	0.396	6	0.303	0.324	Rio Grande River Data of Nordin and Beverage (1965)
2944.8	43.282	0.262	0.343	5	0.262	0.272	
		0.1192	0.102		0.262	0.233	Vanoni (1965)

Then the field experimental data in which the dimensionless water depth (*H/d*) was approximately 10^4 were collected as the correlation data with the calculated results by the empirical formula. The calculated results and the collected field measurement data were listed in *Table 4*.

It can be seen from *Table 3* that when the sediment concentrations are less than or equal to 5 PPM the calculation results by this empirical formula are in good agreement with the data collected from extant studies generally, i.e., Rio Grande River Data of Nordin and Beverage (1965) and South American River & Canal Data of NEDECO (1973). While the incipient velocity of sediment calculated by this empirical formula is lower than the collected field measurement data when the sediment concentrations are more than 5 PPM and approach 10 PPM, for instance, Red River Data of Toffaleti

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(1968), because the hydrodynamic condition is considered to have exceeded the incipient condition of sediment according to the analysis of sediment transport and the distribution of suspended sediment concentration in the fluid when the sediment particles begin to suspend (Ted et al., 1974), when the sediment concentrations approach 10 PPM. It can be seen from *Table 4* that the empirical formula can work well when it is used to calculate the incipient of granular sediment in the flow with high dimensionless water depth. *Figure 6* depicts that the calculated values calculated by *Equation 22* and the 20% error lines against measured data.

Table 4. *Comparisons with extant experimental data (2)*

H/D	Water depth (*m*)	*d* (*mm*)	ρ_s (*kg/m*^{3})	Extant data (*m/s*)	Calculated velocity (*m/s*)	Data sources
1700.526	0.3231	0.19	1.3	0.24	0.231	Simons, Bishop and Richardson (1965)
1604.211	0.3048	0.19	1.3	0.232	0.229	
1027.407	0.2774	0.18	1.56	0.255	0.253	
1099.286	0.3078	0.24	1.67	0.293	0.261	
1700.526	0.3241	0.19	2.65	0.24	0.231	Guy et al. (1966)
1492.105	0.2835	0.19	2.65	0.264	0.226	
1460	0.2774	0.19	2.65	0.255	0.225	

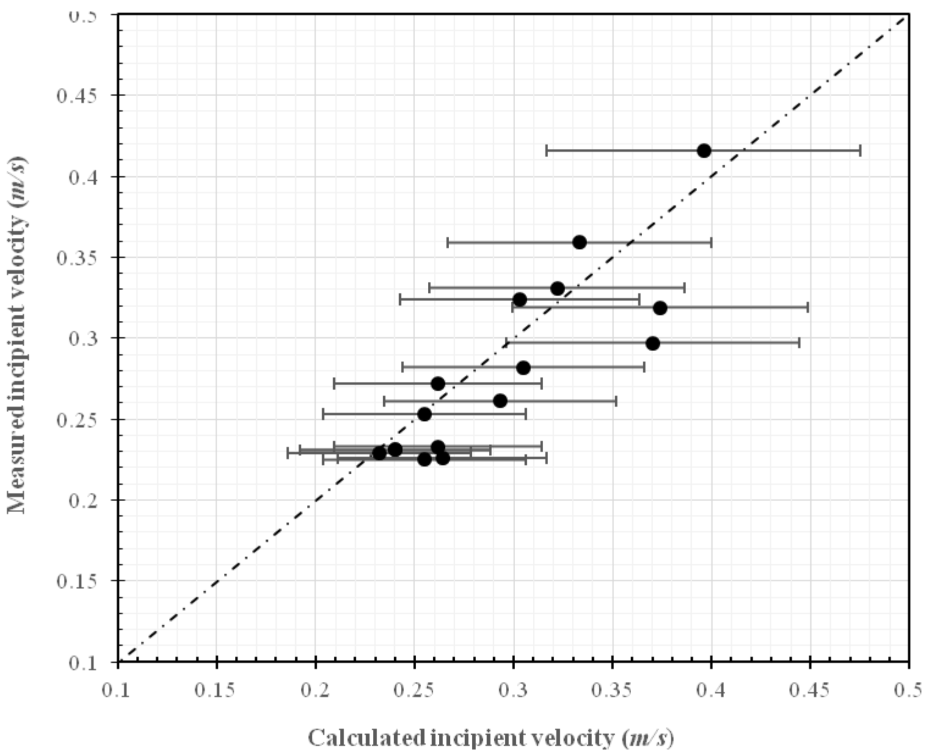

Figure 6. *Comparison between calculated results and measured results*

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Conclusion

In this paper, the incipient condition of the granular sediment in the flow with great dimensionless water depth was studied by the method of flume experiment and data fitting. Firstly, the similarity condition between experimental flow and prototype flow with great dimensionless water depth was analysed firstly. Then based on the above analysis, the boundary layer momentum thickness was introduced instead of water depth due to the limitation of the flume experiments. Subsequently, the flume experiment results and a series of extant data were collected and calculated to derive the formula for the incipient velocity of granular sediment in the flow with great dimensionless water depth. The conclusions of this study can be summarized as follows:

The similarity condition between the experimental flow and the prototype flow with great dimensionless water depth is that the ratio of water depth and the particle size of sediment (*H/d*, which is considered as the dimensionless water depth in this paper) in experimental flow is the same with that in prototype flow with great dimensionless water depth, based on the logarithmic velocity distribution law.

Water depth cannot be used as a direct variable in the formula for the incipient condition of the granular sediment in the flow with great dimensionless water depth due to the limitation of flume experiment, which should be replaced by the boundary layer momentum thickness.

An empirical formula for the incipient velocity for the granular sediment in great dimensionless water depth was proposed by introducing boundary layer momentum thickness as an immediate variable. The formula takes boundary layer momentum thickness, particle size and the proportion of sediment and fluid as the main variables. Compared with the normative formula of Wuhan Institute of Hydraulic and Electric Engineering, the increasing trend of the incipient velocities of the granular sediments in the flow with great dimensionless water depth calculated by the two formulas are similar with each other when $H/d \leq 1 \times 10^4$, and all increase slowly, but when $H/d > 1 \times 10^4$, the increasing trend of the incipient velocities of the granular sediments calculated by the formula proposed in this paper is much slower than that by the normative formula of Wuhan Institute of Hydraulic and Electric Engineering. Finally, the accuracy of the proposed formula is verified.

Recommendations for future researches

Firstly, the viscous force between sediment particles should be considered in future researches. Because the sediment particles behave in a cohesive manner when the particle size is less than 500 μm, moreover, the viscous force becomes the main resistance to the initial motion of sediment, and the effect of gravity on the initial motion of sediment can be neglected, when the particle size is less than 60 μm. But there is no convincing result about the incipient condition of cohesive sediment. In future researches, the concept of sediment fluidization can be introduced to study on the incipient conditions of cohesive sediment, and different sediment fluidization level will cause the variation of starting velocity.

Secondly, the application condition and the form of the formula need further study. Because that the research results in this paper are based on logarithmic velocity distribution, which is reliable for the unidirectional flow. However, the applicability of the condition in complex flow field, such as wave or tide, has yet to be verified.

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APPENDIX

Detailed collected data

H/d	**Depth (*m*)**	***d* (*mm*)**	**Concentration (*ppm*)**	**V_c (*m/s*)**	**Data sources**
930.526	0.1768	0.19	1	0.22	Simons, Bishop and Richardson (1965)
705.789	0.1341	0.19	2	0.26	
642.142	0.1798	0.28	1	0.269	
492.766	0.2316	0.47	1.6	0.347	
486.383	0.2286	0.47	2.3	0.353	
480	0.2256	0.47	2.5	0.365	
461.818	0.1524	0.33	3.5	0.319	
350	0.189	0.54	0.5	0.297	Yalin and Karahan (1979) ASCE
1.151	0.0012	1		0.215	
6.276	0.0063	1		0.289	
8.369	0.0047	0.56		0.215	
64.776	0.0065	0.1		0.231	
11.788	0.0047	0.4		0.212	
30.254	0.0057	0.19		0.206	
39.186	0.0055	0.14		0.205	Li and Sun (1965)
821.428	0.115	0.14		0.21	
771.428	0.108	0.14		0.22	
857.142	0.12	0.14		0.2	
28.5	0.057	2		0.45	
30.5	0.061	2		0.46	
38.5	0.077	2		0.47	
49.5	0.099	2		0.49	
48.5	0.097	2		0.49	
61.363	0.054	0.88		0.32	
45.454	0.04	0.88		0.3	
105.263	0.04	0.38		0.24	
115.789	0.044	0.38		0.25	
155.263	0.059	0.38		0.27	
168.421	0.064	0.38		0.24	
215.384	0.056	0.26		0.2	
192.307	0.05	0.26		0.24	
234.615	0.061	0.26		0.21	
188.461	0.049	0.26		0.23	
196.153	0.051	0.26		0.24	
271.428	0.038	0.14		0.19	
264.285	0.037	0.14		0.2	
278.730	0.0878	0.32	0.2	0.243	Govt. of West Bengal (1965)
344.444	0.1085	0.32	0.7	0.271	
391.746	0.1234	0.32	1	0.283	
402.539	0.1268	0.32	1.1	0.288	
427.619	0.1347	0.32	1.1	0.285	
464.444	0.1463	0.32	1.3	0.279	

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480	0.1512	0.32	1.8	0.282	
506.984	0.1597	0.32	2.6	0.294	
13.618	0.0335	2.46	1.6	0.447	Casey (1935)
47.561	0.117	2.46	1.6	0.482	
47.439	0.1167	2.46	0.019	0.454	
290.196	0.148	0.51	0.094	0.288	Costello (1974)
288.054	0.1495	0.52	1.679	0.311	
264.167	0.1585	0.6	0.473	0.32	
239.394	0.158	0.66	0.819	0.324	
194.937	0.154	0.79	0.764	0.357	
2032	0.3048	0.15	0.8	0.181	Davies (1971)
2032	0.3048	0.15	0.9	0.21	
706.316	0.1342	0.19	2	0.258	Guy, Simons and Richardson (1966)
642.143	0.1798	0.28	1	0.269	
663.778	0.2987	0.45	0.7	0.242	
548.667	0.2469	0.45	1.2	0.24	
541.778	0.2438	0.45	0.7	0.242	
392.889	0.1768	0.45	0.7	0.238	
392.667	0.1767	0.45	1.4	0.252	
223.556	0.1006	0.45	1	0.225	
337.527	0.3139	0.93	0.4	0.404	
330.968	0.3078	0.93	0.4	0.455	
492.766	0.2316	0.47	0.4	0.347	
486.383	0.2286	0.47	0.4	0.353	
480	0.2256	0.47	0.4	0.365	
69.521	0.2176	3.13	0.1	0.581	Ho (1939)
83.833	0.2624	3.13	0.45	0.656	
45.271	0.1417	3.13	0.73	0.547	
58.339	0.1826	3.13	1.11	0.602	
67.188	0.2103	3.13	1.55	0.631	
78.690	0.2463	3.13	2.89	0.689	
33.195	0.1039	3.13	0.97	0.478	
45.367	0.142	3.13	0.54	0.59	
50.825	0.2216	4.36	0.089	0.633	
56.491	0.2463	4.36	0.19	0.664	
35.573	0.1551	4.36	0.26	0.61	
42.293	0.1844	4.36	0.66	0.667	
46.559	0.203	4.36	0.97	0.698	
50.183	0.2188	4.36	1.79	0.734	
35.525	0.2231	6.28	0.16	0.724	
16.891	0.1061	6.28	1.12	0.714	
18.203	0.1094	6.01	1.3	0.692	
29.019	0.1213	4.18	1.2	0.65	Mavis et al. (1937)
30.837	0.1289	4.18	1.3	0.695	
31.794	0.1329	4.18	1.3	0.715	
22.608	0.0945	4.18	1.3	0.623	

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24.067	0.1006	4.18	0.8	0.656	
25.383	0.1061	4.18	0.7	0.660	
25.813	0.1079	4.18	1.3	0.658	
26.244	0.1097	4.18	1.2	0.69	
26.459	0.1106	4.18	2.3	0.713	
25.608	0.0799	3.12	0.9	0.531	
26.955	0.0841	3.12	1.1	0.554	
28.045	0.0875	3.12	2.2	0.567	
30.16	0.0941	3.12	1.9	0.568	
30.481	0.0951	3.12	2.8	0.622	
21.186	0.0661	3.12	2.9	0.532	
22.179	0.0692	3.12	2	0.539	
16.410	0.0512	3.12	2.8	0.523	
15.833	0.0494	3.12	2.8	0.552	
33.054	0.0671	2.03	1.3	0.465	
35.419	0.0719	2.03	1.2	0.481	
37.537	0.0762	2.03	3	0.501	
26.552	0.0539	2.03	2	0.46	
28.818	0.0585	2.03	1.8	0.515	
34.61	0.0488	1.41	2.8	0.366	
24.584	0.0917	3.73	1	0.563	
26.971	0.1006	3.73	1.8	0.581	
28.204	0.1052	3.73	2.3	0.624	
30.08	0.1122	3.73	2.9	0.641	
17.48	0.0652	3.73	1.4	0.539	
19.035	0.071	3.73	1.3	0.56	
20.429	0.0762	3.73	2.3	0.603	
32.679	0.0549	1.68	1.9	0.402	
23.75	0.0399	1.68	2.7	0.386	
41.933	0.1128	2.69	1.133	0.497	Meyer-Peter and Muller (1948)
232.308	0.0604	0.26	3	0.127	Mutter (1971)
233.462	0.0607	0.26	2	0.126	
249.615	0.0649	0.26	2	0.143	
1280	0.32	0.25	0.8	0.369	Nordin (1976)
532.456	0.607	1.14	2.9	0.515	
855	0.3078	0.36	1.9	0.36	O'Brien (1936)
868.611	0.3127	0.36	1.9	0.348	
4.324	0.096	22.2	0.071	0.807	Paintal (1971)
4.739	0.1052	22.2	0.074	0.883	
5.149	0.1143	22.2	0.039	0.949	
5.631	0.125	22.2	0.029	0.967	
5.946	0.132	22.2	0.012	1.079	
6.604	0.1466	22.2	0.058	1.057	
6.73	0.1494	22.2	0.155	1.141	
7.14	0.1585	22.2	0.171	1.173	
7.347	0.1631	22.2	0.396	1.235	

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7.69	0.1707	22.2	0.374	1.27
8.239	0.1829	22.2	0.632	1.287
8.401	0.1865	22.2	0.706	1.329
9.131	0.2027	22.2	1.572	1.376
6.455	0.1433	22.2	0.88	1.016
6.455	0.1433	22.2	1.009	0.973
7.14	0.1585	22.2	1.214	1.075
7.414	0.1646	22.2	1.144	1.223
8.306	0.1844	22.2	2.16	1.277
6.455	0.1433	22.2	1.683	0.994
6.28	0.1387	22.2	1.1791	0.849
7.072	0.157	22.2	1.683	0.939
10.201	0.0811	7.95	0.189	0.764
6.742	0.0536	7.95	0.004	0.532
10.767	0.0856	7.95	0.061	0.724
8.516	0.0677	7.95	0.039	0.595
12.264	0.0975	7.95	0.574	0.794
5.522	0.0439	7.95	0.005	0.649
8.201	0.0652	7.95	0.012	0.713
13.421	0.1067	7.95	0.185	0.755
13.648	0.1085	7.95	0.222	0.857
9.623	0.0765	7.95	0.006	0.668
11.27	0.0896	7.95	0.025	0.743
12.956	0.103	7.95	1.1669	0.842
15.409	0.1225	7.95	1.55	0.91
14.566	0.1158	7.95	0.202	0.856
5.786	0.046	7.95	0.007	0.741
7.208	0.0573	7.95	0.006	0.568
6.704	0.0533	7.95	0.001	0.535
5.371	0.0427	7.95	0.026	0.624
5.824	0.0463	7.95	0.01	0.602
6.138	0.0488	7.95	0.008	0.635
6.667	0.053	7.95	0.011	0.585
3.61	0.0287	7.95	0.001	0.54
5.522	0.0439	7.95	0.066	0.692
11.597	0.0922	7.95	0.003	0.504
10.239	0.0814	7.95	0.004	0.495
9.925	0.0789	7.95	0	0.393
10.893	0.0866	7.95	0.001	0.483
12.654	0.1006	7.95	0.005	0.493
43.64	0.1091	2.5	2.5	0.591
38.16	0.0954	2.5	0.192	0.497
33.04	0.0826	2.5	0.032	0.388
40.24	0.1006	2.5	0.686	0.659
32.92	0.0823	2.5	0.092	0.508
16.84	0.0421	2.5	0.002	0.419

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19.88	0.0497	2.5	0.01	0.405	
22.8	0.057	2.5	0.014	0.435	
28.04	0.0701	2.5	0.387	0.442	
16.84	0.0421	2.5	0.002	0.443	
24.28	0.0607	2.5	0.01	0.408	
26.84	0.0671	2.5	0.02	0.554	
33.28	0.0832	2.5	0.463	0.506	
26.84	0.0671	2.5	0.266	0.425	
18.4	0.046	2.5	0.012	0.404	
23.04	0.0576	2.5	0.145	0.484	
15.72	0.0393	2.5	0.002	0.441	
637.656	0.3048	0.478	0.77	0.283	Pratt (1970)
637.659	0.3048	0.478	0.506	0.267	
637.656	0.3048	0.478	0.28	0.248	
637.656	0.3048	0.478	1.37	0.311	
637.656	0.3048	0.478	2.9	0.338	
318.828	0.1524	0.478	2.89	0.272	
318.828	0.1524	0.478	1.54	0.253	
318.828	0.1524	0.478	0.582	0.236	
318.828	0.1524	0.478	0.278	0.217	
956.485	0.4572	0.478	0.12	0.247	
956.485	0.4572	0.478	0.262	0.259	
956.485	0.4572	0.478	0.146	0.271	
956.485	0.4572	0.478	0.307	0.283	
956.485	0.4572	0.478	0.632	0.304	
956.485	0.4572	0.478	1.73	0.336	
283.72	0.061	0.215	0.184	0.229	Taylor (1971）
283.72	0.061	0.215	0.737	0.229	
283.72	0.061	0.215	0.049	0.213	
283.72	0.061	0.215	0.143	0.213	
283.72	0.061	0.215	0.003	0.198	
283.72	0.061	0.215	0.01	0.198	
21.423	0.0602	2.81	0.215	0.502	
21.423	0.0602	2.81	0.051	0.502	
17.864	0.0502	2.81	1.056	0.638	
21.423	0.0602	2.81	0.549	0.532	
21.423	0.0602	2.81	0.05	0.487	
21.423	0.0602	2.81	0.023	0.487	
169.747	0.0606	0.357	0.047	0.229	
317.277	0.0606	0.191	1.65	0.229	
244.354	0.0606	0.248	0.025	0.204	
57.009	0.061	1.07	0.053	0.305	
57.009	0.061	1.07	0.036	0.305	
57.009	0.061	1.07	2.06	0.335	
57.009	0.061	1.07	1.125	0.335	
57.009	0.061	1.07	0.423	0.32	

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57.009	0.061	1.07	0.255	0.321	
750	0.3	0.4		0.326	Bai et al. (2005)
272.727	0.3	1.1		0.368	
230.769	0.3	1.3		0.398	
2.075	0.083	40	0.108	1.2	Yang et al. (2006)
2.257	0.0903	40	0.648	1.44	
3.49	0.1396	40	4.296	1.79	
5.533	0.083	15		0.964	
4.667	0.035	7.5		0.686	
5.04	0.126	25		1.19	
5.075	0.203	40		1.478	
6	0.021	3.5		0.571	
6	0.009	1.5		0.333	
2.225	0.089	40		1.348	
2.56	0.064	25		1.094	
3	0.021	7		0.714	
2.667	0.04	15		0.875	
2.857	0.01	3.5		0.4	
7.2	0.054	7.5		0.556	
7.714	0.027	3.5		0.444	
10	0.015	1.5		0.333	
7.333	0.11	15		0.864	
8.16	0.204	25		1.201	
3.067	0.023	7.5		0.565	
3.428	0.012	3.5		0.417	
2.867	0.043	15		0.814	
3.28	0.082	25		1.159	
2.975	0.119	40		1.345	

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